Optimal Menu of Menus with Self-Control Preferences*

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September 12, 2005

Abstract

Standard theories of optimal pricing are based on the assumption that consumers are free from temptation. To study whether the presence of temptation has a significant impact on a firm's optimal pricing strategy, we consider the formulation of temptation due to Gul and Pesendorfer (2001) and study the monopolist's nonlinear pricing problem. We show that if temptation raises consumers' marginal value for the quality of goods, the firm can achieve perfect discrimination, by offering multiple menus that target different consumer types. To eliminate consumers' incentives to mimic other types, the firm adds to the menus items that are tempting and ex ante undesirable for unintended customers. The perfect discrimination result is robust, holding even if the deviation from standard preferences is arbitrarily small, and extending to other behavioral formulations. We also show that participation fees, which play little role in the standard problem, have an effect of reducing consumers' disutility from self-control and enable the firm to extract more surplus.

JEL Classification: D11, D42, D82, L12, L15, M31
Keywords: Temptation, self-control, commitment, nonlinear pricing, second-degree price discrimination, perfect discrimination, entry fees, specialization.

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*The first version: November 4, 2004.

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1 Introduction

Standard theories of firms' pricing behavior are based on the assumption that consumers are free from temptation and have complete control of what they choose. Under the assumption, consumers can easily stop smoking, overeating, overspending, wasting time, etc. Real consumers, on the other hand, have temptation and incomplete self-control. Assuming these features away may not be a reasonable approximation of the average consumer, and there is a recent theoretical advance that allows one to formulate the behavior of consumers with temptation. The present paper uses the formulation due to Gul and Pesendorfer (2001) and studies a firm's optimal selling strategy in the presence of temptation.

Standard economics assumes that a consumer evaluates a set of alternatives based only on the most preferred element in the set, since it is what he chooses. This implies that if a set X contains another set Y, he likes X at least as well as Y: there is no disutility of having more options. However, consumers who are subject to temptation may dislike a larger choice set since it may contain options that are tempting and ex ante undesirable. For example, suppose that there are two options, s (salad) and b (burger), and a consumer prefers s to b but b is tempting (whatever this means). Then, the consumer may prefer not to have b in the choice set. The reason is that confronted with the choice set $\{s,b\}$, he may succumb to temptation and choose a burger, or even if he resists temptation and chooses a salad, he may incur a psychological cost in the process of exercising self-control. The set $\{s\}$ is desirable for the consumer since it gives no room for temptation, and allows him to commit to the ex ante desirable choice.

To formulate choice behavior of this kind, Gul and Pesendorfer (2001) introduced the following class of preferences: a consumer prefers a choice set X to another set Y if and only if $W(X) > W(Y)$, where W is defined by

$$W(X) \equiv \max_{x \in X} \Big[U(x) + V(x) \Big] - \max_{x \in X} V(x).$$

The interpretation is that U represents preferences that the consumer would like to commit to, while V is the utility function that represents his temptation. The first maximization identifies what the consumer actually chooses as a compromise between these utility functions. The "bargaining power" of each utility function—the strength of self-control—is determined by the relative scales of the functions. The second maximization identifies the *most tempting* alternative in X. To understand the formulation, rearrange $W(X)$ as

$$W(X) = U(\hat{x}) - \Big[\max_{x \in X} V(x) - V(\hat{x}) \Big],$$

where $\hat{x}$ is a maximizer of $U(x)+V(x)$. The term in square brackets is the forgone utility from exercising self-control: the tempted part of the consumer wants to choose a maximizer of $V(x)$ but ends up with $\hat{x}$ after self-control. This forgone utility is interpreted as the cost of self-control. If $U = V$, there is no temptation and preferences are standard.

This paper considers consumers who have preferences of the type described above and studies a seller's optimal pricing decision. We use the classic model of nonlinear pricing by Mussa and Rosen (1978) and Maskin and Riley (1984) and consider a monopolist selling goods that are indexed by a single-dimensional quality level $q \in \mathbb{R}_+$. The seller does not observe consumers' preferences directly and therefore can set prices only via indirect price discrimination schemes that rely on consumers' self-selection.

In the standard nonlinear pricing problem, the seller chooses a set of goods $Q \subseteq \mathbb{R}_+$ to sell and a price function $p\colon Q \to \mathbb{R}_+$ that specifies a price $p(q)$ for each quality level. The choice of Q and p determines the choice set for consumers, which is a menu of quality-price pairs given by $M = \{(q, p(q)) : q \in Q\}$. Given this menu, each consumer chooses a most preferred pair $(q, p(q))$. Anticipating consumers' choices, the seller chooses a menu that maximizes expected profits.

The present paper extends the pricing problem to the case in which consumers exhibit temptation as described above. In doing so, we also allow the seller to offer multiple menus. While the number of menus is immaterial in the standard setting, it matters in the present setting. Since consumers may prefer smaller, less tempting menus, the seller may profit from offering multiple menus, i.e., a menu of menus. For example, the seller may open multiple retail stores (possibly with different brand names) with smaller, specialized selections and let consumers choose which store to visit. In this way, the seller can make the selection in each store less tempting and hence more appealing to consumers with costly self-control.

Another example where a seller offers a menu of menus is weight-loss programs (e.g., Weight Watchers). They typically offer a variety of plans, specifying the aimed weight loss, the number of weekly visits, the amount of food discount, the penalty fee for weight gain, etc. Each plan is a menu M specifying the total fee $p_M(q)$ for each realized level q of weight loss. Given a plan, the participant optimally chooses a weight-loss level subject to temptation and the cost of self-control. Anticipating the optimal weight-loss choice within each plan, each consumer chooses a plan that maximizes his expected utility.

Since consumers choose a menu and then an item in the menu, the seller's problem has to deal with new conditions of individual rationality and incentive compatibility that pertain to the choice of a menu, as well as the usual conditions of individual rationality and incentive compatibility that pertain to the choice within the chosen menu.

We study a simple case in which there are two types of consumers: high and low. As usual, the high-type consumers have a higher marginal value for quality than the low types. Nonetheless, the statement is ambiguous in the present context since a consumer can be thought of as having multiple states of mind and his marginal value depends on the state. Each consumer has three possible states: (i) *untempted state*, which is captured by the utility function U and in which the consumer is not tempted; (ii) *tempted state*, which is captured by the utility function V and in which he is succumbing to temptation; and (iii) *self-controlled state*, which is captured by $U+V$ and in which he is tempted but exercising self-control. We assume that a high-type consumer in a given state has a higher marginal value for quality than a low-type consumer in the same state. That is, a change of state does not reverse the relative positions of different types.

There remain a number of cases to consider, since each consumer is characterized by two utility functions. One way to classify these possibilities is to look at the relation between the two utility functions of a given consumer, which pertains to the direction in which the consumer is tempted. We say that a consumer has *upward* temptation if his marginal value for quality is higher when he is tempted than when he is not, which means that he is tempted toward goods of higher quality. Conversely, a consumer has *downward* temptation if his marginal value for quality is lower under temptation. While upward temptation may be easier to imagine, downward temptation is not unusual. For example, in the case of weight-loss programs, downward temptation means that consumers are tempted to lose less weight. Even in the case of shopping, consumers may become more frugal when they make decisions and pay.[1]

Our main result says that if the high-type consumers have upward temptation, the seller can achieve perfect price discrimination. Although each consumer's type is unobservable, the seller can design menus that achieve the same outcome as in the complete information case. This is achieved by offering a menu for each type and "decorating" the one intended for the low types by adding a quality-price pair that is irrelevant for the low-type consumers but is ex post tempting and ex ante undesirable for the high types. Although the added item is not chosen by any consumer, it lowers the high types' ex-ante utility from the menu intended for the low types, thereby eliminating the high types' incentive to mimic the low types.

An example is a restaurant that offers both a buffet and à la carte menu. A consumer who is not tempted may choose the buffet. A consumer who faces upward temptation, on the other hand, may anticipate the temptation to eat too much with the buffet and chooses from the menu as a way to commit. Although the menu is typically more expensive than the buffet (and items often overlap), the menu is optimal for the consumer since he saves on self-control cost and avoids giving in to temptation. Thus, the feasibility of eating more with a buffet serves as a deterrent for the consumer, and this in turn allows the seller to raise the prices for the offerings in the menu.

The perfect discrimination result is robust to a number of modifications of the framework. For example, the result continues to hold if there are more than two types of consumers. The result also extends to the case where the temptation preferences V are stochastic.

Another important observation is that perfect discrimination is possible even if the departure from standard preferences is arbitrarily small. What matters for the result is that the marginal value for quality is higher for the temptation preferences (V) than for the commitment preferences (U), and the magnitude of the difference is immaterial. As V converges to U, the effect of temptation goes to zero and preferences become standard in the limit. However, the optimal schedule does not converge to the one in the standard model, since perfect discrimination is possible except in the limit. This observation suggests that the standard theory of nonlinear pricing is not robust to the introduction of temptation.

If the high types' temptation is downward and therefore temptation lowers their marginal value for quality, then perfect discrimination is not possible. The seller's optimal strategy depends on the degree of the high types' temptation. We say that the high types' temptation is *strong* if their marginal value in the tempted state is lower than that of the low types in the self-controlled state. That is, if the high types are in the tempted state and the low types are in the self-controlled state, the relative positions of the types are reversed. If the reversal does not occur, i.e., if the high types' temptation is *weak*, then we show that offering a single menu is never optimal for the seller. Under weak temptation, the item offered to the low types is tempting to the high types. Thus, by offering to the high types a separate menu that excludes the tempting choice, the seller can reduce the high types' self-control cost and increase their ex-ante utility. This, in turn, allows the seller to extract more surplus from the high types by raising prices. On the other hand, if the reversal occurs, i.e.,

[1] Using survey techniques, Ameriks, Caplin, Leahy, and Tyler (2003) find evidence of heterogeneity in the direction of temptation in a two-period saving problem among TIAA-CREF participants: 20% of participants are tempted to consume less in the first period.

if the high types' temptation is strong, then the seller gains nothing from offering multiple menus. We show that the optimal scheme in this case coincides with the one in the standard nonlinear pricing problem with consumers' preferences given by $U+V$.

In our basic model, the seller is not allowed to charge *entry fees*, i.e., fees that consumers pay even when they do not buy any good or use any service (e.g., membership fees). Entry fees are commonly observed for a number of services, such as gym clubs, cellular phone plans, and weight-loss programs. These services are also examples that appear in the literature of behavioral anomalies. In Section 4, we extend the basic model to allow for entry fees.

An important observation is that entry fees play little role in the standard theory of nonlinear pricing. In the standard model, the participation constraint implies that each consumer's choice must be at least as good as $(0,0)$, and hence whether the seller can charge a positive price for $q=0$ (an entry fee) has no effect on the optimal menu or the profit. However, in the world with temptation, the entry fees can increase profits strictly. This is because entry fees can decrease temptation for consumers whose main temptation is to exit without any purchase (or quit the weight-loss program). Positive entry fees make the option of exiting more expensive and hence less tempting. This reduces consumers' self-control costs and increases their ex-ante utilities, which in turn allows the seller to charge higher prices. The result also implies that if the firm has a strict incentive to charge a positive entry fee for a menu, then the target consumers must have downward temptation.

There is a growing number of papers that study optimal strategies against agents who have non-standard preferences. O'Donoghue and Rabin (1999), Gilpatric (2001), and DellaVigna and Malmendier (2004) study optimal contracts when agents have (quasi-) hyperbolic discounting. Eliaz and Spiegler (2004) derive the optimal contract when the principal knows that agents' preferences change in the second period but agents themselves believe that the change may not occur. Esteban, Miyagawa, and Shum (2003) consider the same problem as the present paper but examine the case in which the seller offers a single menu and there is a continuum of types. Esteban and Miyagawa (2005) extend the model to oligopoly and characterize Nash equilibria when firms compete by offering a menu of menus.[2]

There is also an empirical literature that tests for preference reversals with pricing data. For example, Wertenbroch (1998) finds evidence that consumers tend to forgo quantity discounts for goods that have delayed negative effects (e.g., cigarettes). Della Vigna and Malmendier (2002) find evidence of time inconsistent behavior in consumers' enrollment decisions in health clubs. Miravete (2003) looks for evidence of irrational behavior in consumers' choices of calling plans and finds that their behavior is consistent with rationality and learning. Oster and Morton (2004) find evidence that magazines that have a payoff in the future (e.g., intellectual magazines) are sold at a higher price.

There are a few papers that also prove perfect discrimination in the context of nonlinear pricing. Hamilton and Slutsky (2004) show that if the number of consumers of each type is finite and common knowledge among the seller and consumers, the seller can use a type-revelation mechanism to achieve perfect discrimination. The mechanism offers a menu so that truth-telling is a

[2]Gul–Pesendorfer preferences have been applied to a variety of contexts: e.g., Krusell, Kuruşcu, and Smith (2000) to a neoclassical growth model; Krusell, Kuruşcu, and Smith (2002) and DeJong and Ripoll (2003) to an asset-pricing problem; and Miao (2004) to an optimal stopping problem.

dominant action for the low types. Given this, if a single high-type consumer misrepresents his type, the seller knows that someone lied. The seller then punishes all consumers who announced themselves as the low type by giving them a trivial choice. Bagnoli, Salant, and Swierzbinski (1995) consider a dynamic setting with complete information about preferences and the number of consumers, and show that if the time horizon is infinite and the common discount factor is sufficiently close to one, there is a subgame-perfect equilibrium where the seller achieves perfect discrimination. Unlike these papers, we use the same informational structure as the standard model of nonlinear pricing. We instead perturb consumers' preferences to have temptation and show that even if the perturbation is arbitrarily small, the standard result collapses and the seller can achieve perfect discrimination.

2 Model

The model is a standard model of nonlinear pricing except that consumers' preferences exhibit temptation.[3] The seller is a monopolist offering a collection of goods. Each good is indexed by a number $q \in \mathbb{R}_+$, which, as usual, represents either quality or quantity. (Throughout the paper, however, q is referred to as quality.) The good with $q = 0$ is the equivalent of nothing. Each consumer is interested in buying at most one unit of at most one good.

An ***offer*** is a pair $(q,t) \in \mathbb{R}^2_+$. If the offer is available, consumers can buy (one unit of) good q at a total charge of t.

A ***menu*** is a compact set of offers $M \subseteq \mathbb{R}^2_+$ such that $(0,0) \in M$. The restriction that $(0,0) \in M$ represents our assumption that consumers who do not buy any good do not have to pay; this will be relaxed in Section 4. For simplicity, we often specify a menu without noting that it includes $(0,0)$: by writing $M = \{(q,t),(q',t'),\ldots\}$, we mean $M = \{(0,0),(q,t),(q',t'),\ldots\}$.

2.1 Consumers

There are two types of consumers, high and low, denoted by H and L, respectively. The type of a consumer is not observable to the seller. A generic type is denoted by γ. We let $n_\gamma \in (0,1)$ denote the fraction of consumers of type γ, with $n_L + n_H = 1$.

Consumers have preferences over menus. Using the utility representation by Gul and Pesendorfer (2001), we assume that the utility function of a consumer of type γ is given by

$$W_\gamma(M) \equiv \max_{(q,t)\in M} \big[U_\gamma(q,t) + V_\gamma(q,t)\big] - \max_{(q,t)\in M} V_\gamma(q,t), \tag{1}$$

where U_γ and V_γ are functions from $\mathbb{R}^2_+$ to $\mathbb{R}$. Function U_γ represents the preferences that the consumer would like to commit to, while V_γ represents his temptation. The offer that the consumer actually chooses is one that maximizes the utility function $U_\gamma + V_\gamma$, which is interpreted as the preference relation resulting from the exercise of self-control. The relative power of each preference ranking in determining the choice from a menu depends on the relative scale of U_γ and V_γ. For example, increasing the relative scale of U_γ (e.g., by multiplying it by a scaler $\alpha > 1$) increases the consumer's "willpower." We call $U_\gamma + V_\gamma$ the ***ex-post utility*** and the associated maximization problem the ***ex-post problem***.

[3] For the standard model of nonlinear pricing, see, e.g., Salanié (1998) and Fudenberg and Tirole (1992, Chapter 7).

The solution to the second maximization problem in (1) represents the most tempting choice in the menu. The role of this maximization can be seen by rearranging (1) into

$$W_\gamma(M) = U_\gamma(q_\gamma, t_\gamma) - \Big[\max_{(q,t)\in M} V_\gamma(q,t) - V_\gamma(q_\gamma, t_\gamma)\Big],$$

where (q_γ, t_γ) denotes a maximizer of $U_\gamma + V_\gamma$. The term in square brackets measures the utility that the consumer loses from self-control: the tempted part of the consumer would like to maximize V_γ but ends up with (q_γ, t_γ) after self-control. This forgone utility quantifies his disutility from self-control and is called the ***self-control cost***. Then, the overall utility from choosing the menu M equals $U_\gamma(q_\gamma, t_\gamma)$ minus the self-control cost. We call the overall utility the ***ex-ante utility***.

Note that the second maximization in (1) is irrelevant for the consumer's optimal choice from the menu, since the choice is determined by the first maximization. The second maximization matters when the consumer chooses a menu. A menu that contains tempting items, those that give high values for the second maximization, give a low ex-ante utility. This makes it possible for consumers to prefer a smaller menu. However, consumers do not necessarily prefer a smaller menu since it may also attain a lower value in the first maximization.

The utility functions satisfy the following standard assumptions:

Assumption 1. For each $\gamma \in \{L, H\}$, U_γ and V_γ are continuous, strictly increasing in q, strictly decreasing in t, quasi-concave, and satisfy $U_\gamma(0,0) = V_\gamma(0,0) = 0$.

A convenient way to compute the ex-ante utility $W_\gamma(M)$ is to identify $y \in \mathbb{R}^2_+$ such that

$$U_\gamma(y) + V_\gamma(y) = \max_{x\in M}\Big[U_\gamma(x) + V_\gamma(x)\Big],$$
$$V_\gamma(y) = \max_{x\in M} V_\gamma(x).$$

Graphically, y is the intersection of the highest-utility indifference curves of $U_\gamma + V_\gamma$ and V_γ given M. The offer y is useful since $W_\gamma(M) = U_\gamma(y)$.

To state our additional assumptions, we introduce a set of binary relations between utility functions. Given two utility functions U and $\hat{U}$ (defined over $\mathbb{R}^2_+$), we write $U \succsim \hat{U}$ if at any point $(q,t) \in \mathbb{R}^2_+$, the indifference curve of U is at least as steep as that of $\hat{U}$ if we measure the first (resp. second) argument on the horizontal (resp. vertical) axis. Formally, $U \succsim \hat{U}$ if and only if for all $(q,t), (q',t') \in \mathbb{R}^2_+$ such that $q' > q$,

$$\hat{U}(q',t') \geq \hat{U}(q,t) \text{ implies } U(q',t') \geq U(q,t), \text{ and}$$
$$\hat{U}(q',t') > \hat{U}(q,t) \text{ implies } U(q',t') > U(q,t).$$

If $U \succsim \hat{U}$ and $\hat{U} \succsim U$, these functions have identical ordinal preferences, which we denote as $U \sim \hat{U}$.

We also write $U \succ \hat{U}$ if the indifference curve of U is strictly steeper than that of $\hat{U}$ at any $(q,t) \in \mathbb{R}^2_+$. Formally, $U \succ \hat{U}$ if and only if for all $(q,t), (q',t') \in \mathbb{R}^2_+$ such that $q' > q$,

$$\hat{U}(q',t') \geq \hat{U}(q,t) \text{ implies } U(q',t') > U(q,t).$$

We place the following assumptions on U_γ and V_γ.

Assumption 2. $U_H \succ U_L$, $V_H \succ V_L$, and $U_H + V_H \succ U_L + V_L$.

Assumption 3. For each $\gamma \in \{L, H\}$, either $V_\gamma \succ U_\gamma$ or $V_\gamma \prec U_\gamma$.

Assumption 4. For any pair of utility functions $f, g \in \{U_\gamma, V_\gamma, U_\gamma + V_\gamma : \gamma \in \{L, H\}\}$ such that $f \prec g$ and any pair of offers $x, y \in \mathbb{R}^2_+$ such that $f(x) > f(y)$, there exists an offer $z \in \mathbb{R}^2_+$ such that $f(z) = f(y)$ and $g(z) = g(x)$.

Assumption 2 is a single-crossing property saying that the indifference curves of U_γ, V_γ, and $U_\gamma + V_\gamma$ are steeper (in the strict sense) for the high-type consumers. Thus, the high types have a higher marginal value for quality than the low types in any of the preference relations.[4]

Assumption 3 says that each consumer is tempted in one direction or the other. For γ such that $V_\gamma \succ U_\gamma$ (which implies $V_\gamma \succ U_\gamma + V_\gamma \succ U_\gamma$), temptation raises the marginal value for quality, which means that the consumer is tempted towards goods of higher q. In this case, we say that the consumer has ***upward temptation***. If $V_\gamma \prec U_\gamma$ (which implies $V_\gamma \prec U_\gamma + V_\gamma \prec U_\gamma$), temptation lowers the marginal value for quality, and hence the consumer has ***downward temptation***.

Assumption 4 says that if utility function g has steeper indifference curves than f, then for any indifference curve of f and any point x with a higher value of f, the indifference curve of g through x crosses the indifference curve of f somewhere. Thus, two distinct utility functions do not have indifference curves that get closer and closer to each other but never cross.

2.2 Seller

The seller's problem is to choose a set of menus that maximizes expected profits. The seller can offer any number of menus. But since only two types of consumers exist, we can assume, without loss of generality, that the seller offers at most two menus. Let M_L and M_H denote the pair of menus offered by the seller, where M_H is intended for the high-type consumers and M_L is intended for the low types. A consumer who is not choosing any of these menus is considered to be choosing the trivial menu $M_0 \equiv \{(0,0)\}$. We allow for $M_L = M_H$, in which case the seller is offering a single menu.

Let $x_H = (q_H, t_H)$ denote the offer that the high-type consumers are expected to choose. Since they are expected to choose M_H, we require $x_H \in M_H$. Similarly, let $x_L = (q_L, t_L)$ denote the offer that the low types are expected to choose, and thus $x_L \in M_L$. Formally, let a ***schedule*** be an ordered list (M_L, x_L, M_H, x_H) in which M_H and M_L are menus with $x_L \in M_L$ and $x_H \in M_H$. Given a schedule (M_L, x_L, M_H, x_H), we say that M_γ is ***decorated*** if $M_\gamma \supsetneq \{x_\gamma\}$ (recall that by $\{x_\gamma\}$ we mean $\{x_\gamma, (0,0)\}$).

We let $C(q)$ denote the per-consumer cost of producing good q. For a given (q,t), let $\pi(q,t) \equiv t - C(q)$ denote the per-consumer profit from the offer. We place the following assumptions:

Assumption 5. C is differentiable, strictly increasing, convex, and satisfies $C(0) = 0$.

[4]The third relation in Assumption 2 does not follow from the first two since the first two relations are about ordinal preferences of each utility function and say nothing about the relative scale of U_γ and V_γ for each type. For example, if $V_H \succ V_L \succ U_H \succ U_L$ and the scale of V_γ is small for H and large for L, then $U_L + V_L \succ U_H + V_H$ is possible.

Assumption 6. For any type γ, any utility function $f_\gamma \in \{U_\gamma, V_\gamma, U_\gamma + V_\gamma\}$, and any number $k \in \mathbb{R}$, an offer x that maximizes $\pi(x)$ subject to $f_\gamma(x) \geq k$ exists uniquely and satisfies $x \gg 0$.

The seller's problem is to choose a schedule (M_L, x_L, M_H, x_H) that maximizes

$$n_L \pi(x_L) + n_H \pi(x_H) \tag{2}$$

subject to, for all $\gamma \in \{L, H\}$,

$$W_\gamma(M_\gamma) \geq 0 \qquad (= W_\gamma(\{(0,0)\})), \tag{ex-ante IR}$$
$$W_\gamma(M_\gamma) \geq W_\gamma(M_s) \quad \text{for all } s \in \{L, H\}, \tag{ex-ante IC}$$
$$U_\gamma(x_\gamma) + V_\gamma(x_\gamma) \geq 0 \qquad (= U_\gamma(0,0) + V_\gamma(0,0)), \tag{ex-post IR}$$
$$U_\gamma(x_\gamma) + V_\gamma(x_\gamma) \geq U_\gamma(x) + V_\gamma(x) \quad \text{for all } x \in M_\gamma. \tag{ex-post IC}$$

The first two conditions ensure that consumers have an incentive to choose the *menu* intended for them, while the last two conditions ensure that consumers also have an incentive to choose the *offer* intended for them. Of these, ex-ante IR comes from our assumption that consumers can choose the trivial menu $M_0 \equiv \{(0,0)\}$ and our normalization of utilities, which implies $W_\gamma(M_0) = 0$. On the other hand, ex-post IR comes from our assumption that every menu contains $(0,0)$ and that the same normalization of utilities implies $U_\gamma(0,0) + V_\gamma(0,0) = 0$.[5]

A ***feasible schedule*** is a schedule (M_L, x_L, M_H, x_H) that satisfies all the above constraints. An ***optimal schedule*** is a feasible schedule that solves the maximization problem.

Notice that the profit in (2) depends only on the offers actually chosen, i.e., x_L and x_H. The implicit assumption is that production costs realize only after a consumer makes a purchase and that there is no cost for listing offers in a menu. This assumption is immaterial in the standard setting since the seller does not gain from listing offers that are not chosen by consumers. However, in the world with temptation, the seller may have an incentive to list offers that are never chosen, if these offers are tempting to consumers and affect their menu choices. On the other hand, the following analysis is robust to the introduction of a small cost for listing offers in menus.[6]

There are two useful benchmark problems. The first one is the standard nonlinear pricing problem with utility functions $U_\gamma + V_\gamma$. In this problem, the seller chooses a pair of offers $\{x_L, x_H\}$ that maximizes its expected profits $n_L \pi(x_L) + n_H \pi(x_H)$ subject to IC and IR: for each $\gamma \in \{L, H\}$,

$$U_\gamma(x_\gamma) + V_\gamma(x_\gamma) \geq U_\gamma(x) + V_\gamma(x) \quad \text{for all } x \in \{x_L, x_H\}, \tag{3}$$
$$U_\gamma(x_\gamma) + V_\gamma(x_\gamma) \geq 0. \tag{4}$$

This problem is called the ***standard problem with utility functions*** $\boldsymbol{U_\gamma + V_\gamma}$. For this problem, it is well known that (i) the binding constraints at the solution are IR for the low types and IC for the high types; and (ii) the solution satisfies "efficiency at the top": the high types' marginal value

[5] Ex-ante IR may appear to imply ex-post IR since the second maximization in the ex-ante utility gives a non-positive value. However, ex-ante IR does not imply ex-post IR since ex-ante IR itself places no restriction on x_γ. What ex-ante IR implies is that any ex-post optimal choice gives a non-negative $U_\gamma + V_\gamma$ utility. Ex-post IR and IC ensure that an ex-post optimal choice is x_γ.

[6] Another implicit assumption in the above formulation is that it is also costless for the seller to add *menus*. However, introducing a small cost of creating menus (e.g., setup costs for retail stores) does not affect the results.

equals the marginal cost.

Another useful benchmark is the case where consumers have no temptation: $V_\gamma \sim U_\gamma$ (e.g., $V_\gamma = U_\gamma$). In this case, the ex-ante utility is $W_\gamma(M) = \max_{x \in M} U_\gamma(x)$. Thus, the seller's problem reduces to the standard problem with utility functions U_γ, where U_γ replaces $U_\gamma + V_\gamma$ in (3) and (4). Accordingly, the problem is called the ***standard problem with utility functions*** $\boldsymbol{U_\gamma}$.

2.3 Complete Information

As usual, it is useful to start with the case in which the seller can observe each consumer's type. Suppose that the seller can offer a personalized menu M_γ to a consumer of type γ. If the seller expects that the consumer chooses an offer $x_\gamma \in M_\gamma$, the pair (M_γ, x_γ) needs to satisfy ex-post IR–IC and ex-ante IR, i.e.,

$$U_\gamma(x_\gamma) + V_\gamma(x_\gamma) \geq 0, \tag{5}$$

$$U_\gamma(x_\gamma) + V_\gamma(x_\gamma) \geq U_\gamma(y) + V_\gamma(y) \quad \text{for all } y \in M_\gamma, \tag{6}$$

$$W_\gamma(M_\gamma) \geq 0. \tag{7}$$

However, if (M_γ, x_γ) satisfies (5)–(7), so does $(\{x_\gamma\}, x_\gamma)$. Thus, the seller has no incentive to offer $M_\gamma \supsetneq \{x_\gamma\}$. Then, for a menu $M_\gamma = \{x_\gamma\}$, (6) is vacuous and (7) reduces to

$$U_\gamma(x_\gamma) + V_\gamma(x_\gamma) - \max\{0, V_\gamma(x_\gamma)\} \geq 0. \tag{8}$$

But, since (8) implies (5), the seller's optimal strategy is to offer a menu $\{x^*_\gamma\}$ where x^*_γ maximizes $\pi(x)$ subject to (8). Thus, at the optimum,

$$U_\gamma(x^*_\gamma) + V_\gamma(x^*_\gamma) - \max\{0, V_\gamma(x^*_\gamma)\} = 0. \tag{9}$$

We call x^*_γ a ***perfect discrimination offer*** for type γ.

The characterization of x^*_γ depends on the direction of temptation. If the consumer has upward temptation, (5) implies $V_\gamma(x^*_\gamma) \geq 0$ and thus (9) reduces to $U_\gamma(x^*_\gamma) = 0$. Therefore, x^*_γ is the offer that maximizes $\pi(x)$ subject to $U_\gamma(x) = 0$. By Assumption 6, x^*_γ is unique and $x^*_\gamma \gg 0$. Since $\max\{0, V_\gamma(x^*_\gamma)\} - V_\gamma(x^*_\gamma) = 0$, the menu $\{x^*_\gamma\}$ generates no self-control cost: it is in the seller's interest not to make consumers suffer from self-control.

For consumers with downward temptation, if (5) holds with equality, then $V_\gamma(x^*_\gamma) \leq 0$ and thus (9) holds. Therefore, x^*_γ is the offer that maximizes $\pi(x)$ subject to $U_\gamma(x) + V_\gamma(x) = 0$. Again, x^*_γ is unique and $x^*_\gamma \gg 0$. Since $x^*_\gamma \gg 0$, we have $V_\gamma(x^*_\gamma) < 0$, which implies that the consumer incurs a positive self-control cost equal to $-V_\gamma(x^*_\gamma) > 0$: the consumer is tempted by $(0,0)$ but the seller cannot remove the choice from the menu.

To summarize, x^*_γ maximizes $\pi(x)$ subject to $F_\gamma(x) = 0$, where F_γ is defined by

$$F_\gamma \equiv \begin{cases} U_\gamma & \text{if } U_\gamma \prec U_\gamma + V_\gamma, \\ U_\gamma + V_\gamma & \text{if } U_\gamma \succ U_\gamma + V_\gamma. \end{cases}$$

We conclude this section with a useful characterization of the incentive constraints for the problem with complete information.

Lemma 1. *For any type γ,*

(i) if a pair (M_γ, x_γ) satisfies ex-ante IR and ex-post IR–IC, then $F_\gamma(x_\gamma) \geq 0$;

(ii) if a pair (M_γ, x_γ) satisfies ex-ante IR and ex-post IR–IC, and $F_\gamma(x_\gamma) = 0$, then $W_\gamma(M_\gamma) = 0$;

(iii) if an offer x_γ satisfies $F_\gamma(x_\gamma) \geq 0$, then $(\{x_\gamma\}, x_\gamma)$ satisfies ex-ante IR and ex-post IR.

Proof. If (M_γ, x_γ) satisfies ex-ante IR and ex-post IR–IC,

$$\begin{aligned} 0 \leq W_\gamma(M_\gamma) &= U_\gamma(x_\gamma) + V_\gamma(x_\gamma) - \max_{y \in M_\gamma} V_\gamma(y) \\ &\leq U_\gamma(x_\gamma) + V_\gamma(x_\gamma) - \max\{0, V_\gamma(x_\gamma)\} = \min\{U_\gamma(x_\gamma) + V_\gamma(x_\gamma), U_\gamma(x_\gamma)\}. \end{aligned} \tag{10}$$

The last line is non-negative if and only if $F_\gamma(x_\gamma) \geq 0$, which proves (i) and (ii).[7] For (iii), $F_\gamma(x_\gamma) \geq 0$ implies $U_\gamma(x_\gamma) + V_\gamma(x_\gamma) \geq 0$, which proves ex-post IR. By ex-post IR, $W_\gamma(\{x_\gamma\})$ equals (10), which is non-negative since $F_\gamma(x_\gamma) \geq 0$. Q.E.D.

We now turn to the incomplete information case, which is our main focus.

3 Optimal Menu of Menus

3.1 Upward Temptation

The first main result states that if the high-type consumers have upward temptation, the seller can achieve perfect discrimination. That is, there exists a menu of menus that gives each consumer an incentive to choose the perfect discrimination offer for his true type. The result holds regardless of the direction of the low type's temptation.

Proposition 1 (Perfect Discrimination). *If the high-type consumers have upward temptation, there exists a feasible schedule (M_L, x_L, M_H, x_H) such that $x_L = x_L^*$ and $x_H = x_H^*$.*

To get the intuition, see Figure 1a, where the low types have no temptation ($V_L \sim U_L$). Setting $M_L = M_H = \{x_L^*, x_H^*\}$ does not achieve perfect discrimination since the high types will choose x_L^* as $U_H(x_L^*) + V_H(x_L^*) > U_H(x_H^*) + V_H(x_H^*)$. Setting $M_L = \{x_L^*\}$ and $M_H = \{x_H^*\}$ does not work either, since

$$W_H(M_L) = U_H(x_L^*) + V_H(x_L^*) - V_H(x_L^*) = U_H(x_L^*) > 0 = W_H(M_H),$$

and hence the high types will choose M_L.

A way to achieve perfect discrimination is to set $M_L = \{x_L^*, y\}$ and $M_H = \{x_H^*\}$. In this schedule, y is not intended to be chosen by consumers but to tempt the high-type consumers who have chosen M_L. Indeed, y is the most tempting offer in M_L for the high types and hence

$$W_H(M_L) = U_H(y) + V_H(y) - V_H(y) = U_H(y) = 0 = W_H(M_H).$$

The inequality ensures that the high types have an incentive to choose M_H. The incentive can be made strict by moving up y slightly. For the high types, x_L^* is appealing but if they choose M_L, they

[7]Note that $F_\gamma(x)$ is not identical to $\min\{U_\gamma(x), U_\gamma(x) + V_\gamma(x)\}$. For each type, F_γ is identical to either U_γ or $U_\gamma + V_\gamma$, while $\min\{U_\gamma(x), U_\gamma(x) + V_\gamma(x)\}$ is not identical to either of them.

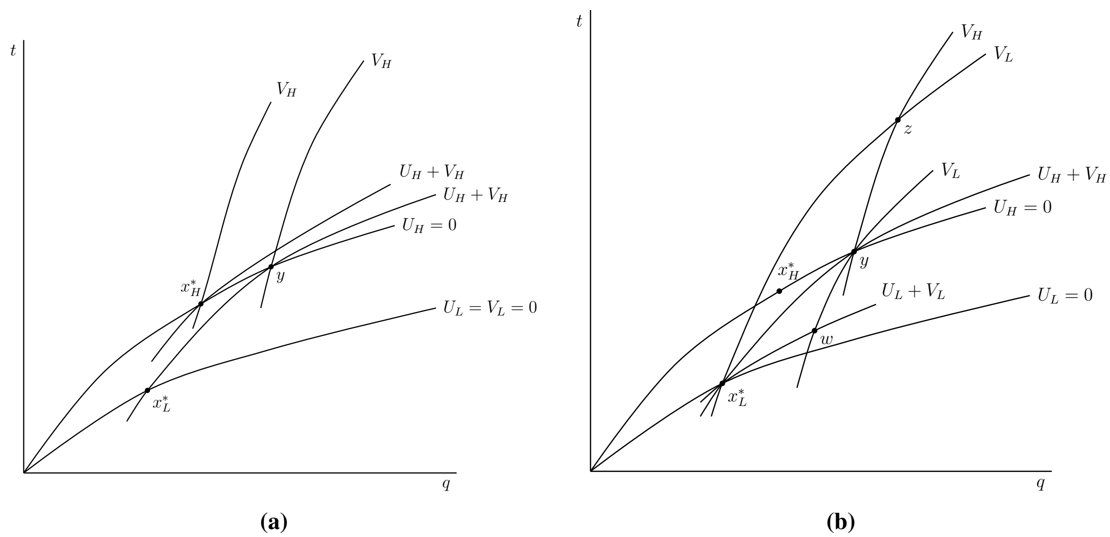


Figure 1: Optimal schedules when the high types have upward temptation. In (a), an optimal schedule offers $M_L = \{x_L^*, y\}$ and $M_H = \{x_H^*\}$. In (b), an optimal schedule offers $M_L = \{x_L^*, z\}$ and $M_H = \{x_H^*\}$.

will be tempted by y. Thus, y works as a deterrent, eliminating the high types' incentive to mimic the low types. It is easy to check that the schedule satisfies the other incentive constraints.

The above choice of y does not necessarily work if the low types also have upward temptation (in particular, $V_L \succ U_H + V_H$), as in Figure 1b. For this preference configuration, offering $M_L = \{x_L^*, y\}$ and $M_H = \{x_H^*\}$ does not work, since y is also the most tempting choice in M_L for the low types and therefore

$$W_L(M_L) = U_L(x_L^*) + V_L(x_L^*) - V_L(y) = U_L(w) < 0.$$

Thus the ex-ante IR is violated. A solution in this case is to offer $M_L = \{x_L^*, z\}$ and $M_H = \{x_H^*\}$. Offer z does not tempt the low types but is as tempting as y for the high types and therefore serves as a deterrent.

The two types of construction in Figure 1 suffice for the general proof (which is given in Appendix A.1).

3.2 Robustness

We here discuss whether Proposition 1 is robust.

Almost-Standard Preferences. The perfect discrimination result holds if $V_H \succ U_H$, and thus, even if V_H is close to U_H. This is interesting because as V_H converges to U_H, temptation goes to zero. Therefore, if V_L also converges to U_L, our model converges to the standard model with utility functions U_γ. However, we have shown that the optimal schedule does not converge to the optimal schedule in the standard problem. By Proposition 1, the seller can achieve perfect discrimination except in the limit. As V_H converges to U_H, the offer y in Figure 1a moves north-east along the $U_H = 0$ curve, but the basic form of the optimal schedule does not change, offering the perfect discrimination offers together with y.

Another feature of the condition $V_H \succ U_H$ is that it depends only on the *ordinal* preferences of the utility functions, making the *scales* of these functions immaterial. Therefore, the perfect discrimination result continues to hold even if we scale down V_H by multiplying it by a small number $\varepsilon > 0$. As $\varepsilon \to 0$, the ordinal preferences of $U_H + V_H$ converge to those of U_H, self-control becomes complete, and the high types' preferences converge to standard preferences with utility function U_H. However, again, even if the low types' preferences also converge to standard preferences, the optimal schedule does not converge to the one in the associated standard problem.[8]

These observations suggest that the standard theory of nonlinear pricing is not robust to the introduction of temptation. The standard optimal schedule is not even an approximation of the optimal schedule once preferences are perturbed to exhibit slight upward temptation.

Many Types. The perfect discrimination result generalizes to the case where more than two types of consumers exist. For any finite number of types, if all types have upward temptation, the seller can achieve perfect discrimination. The basic structure of the proof is the same: menu by menu, the seller adds a decoration that serves as a deterrent for the higher consumer types. Even if only some types have upward temptation, as long as there exists a threshold type $\hat{\gamma}$ such that all $\gamma \geq \hat{\gamma}$ have upward temptation, the seller can still achieve perfect discrimination for all consumers of type $\gamma \geq \hat{\gamma}$ by following the same strategy. (The proofs are in Appendix A.10.)

A Different Model of Preferences with Temptation. The perfect discrimination result is not specific to the Gul–Pesendorfer formulation. The result continues to hold for a different behavioral formulation where consumers have temptation but disregard the cost of self-control. Specifically, suppose that the ex-ante utility is given by $U_\gamma(x_\gamma)$ where $x_\gamma \in M_\gamma$ maximizes $U_\gamma(x) + V_\gamma(x)$. That is, the consumer correctly foresees that he will maximize $U_\gamma + V_\gamma$, but evaluates the outcome with his commitment utility. If preferences are of this form and the high types' temptation is upward, the seller can achieve perfect discrimination by offering $M_L = \{x_L^*, y\}$ and $M_H = \{x_H^*\}$, where y satisfies $U_H(y) + V_H(y) \geq U_H(x_L^*) + V_H(x_L^*)$ and $U_H(y) \leq 0$ (e.g., y in Figure 1a).

Stochastic Temptation. As another robustness check, we can consider the generalizations of Gul and Pesendorfer (2001) proposed by Dekel, Lipman, and Rustichini (2005). A particularly interesting generalization is the one in which temptation preferences are stochastic and consumers are uncertain of which temptation preference will strike them. Specifically, suppose that for each type γ, there is a finite set of states S_γ and that the temptation utility function is V_γ^s in state $s \in S_\gamma$. The state realizes after the consumer chooses a menu. The consumer believes that state $s \in S_\gamma$

[8] The above argument can be made concrete by considering the following parameterization of V_H. Fix a function V^* such that $V^* \succ U_H$ and suppose that V_H is given by

$$V_H(x) = \left[iV^*(x) + (1-i)U_H(x)\right]/w,$$

where $w > 0$ represents the consumer's *willpower* and $i \in [0,1]$ represents the *intensity* of temptation. As $i \to 0$, the ordinal preferences of V_H converge to those of U_H, while w does not affect the ordinal preferences of V_H. Thus, i is an index of how much temptation *can possibly* change preferences (i.e., $U_H \to V_H$). How much temptation *does* indeed change preferences (i.e., $U_H \to U_H + V_H$) is another issue, since the consumer has willpower to resist temptation. As $w \to \infty$, $U_H + V_H$ converges to U_H and hence self-control becomes complete for a fixed i. With this parameterization, Proposition 1 holds for all (w,i) such that $w < \infty$ and $i > 0$. Under the assumption that the low types have no temptation, the standard result applies if and only if either $i = 0$ (no temptation) or $w = \infty$ (complete self-control).

realizes with probability $p_\gamma(s) > 0$. Thus, the ex-ante utility function for a type γ is given by

$$W_\gamma(M) = \sum_{s\in S_\gamma} p_\gamma(s) W_\gamma^s(M), \quad \text{where}$$
$$W_\gamma^s(M) = \max_{x\in M}\left[U_\gamma(x) + V_\gamma^s(x)\right] - \max_{x\in M} V_\gamma^s(x) \quad \text{for all } s \in S_\gamma.$$

Under a weak additional assumption on utility functions, we can show that perfect discrimination is possible if there exists at least one state in which the high-type consumers have upward temptation. Formally, a sufficient condition is that there exists a state $\hat{s} \in S_H$ such that $V_H^{\hat{s}} \succ U_H$ and $V_H^{\hat{s}} \succ V_L^s$ for all $s \in S_L$. The idea is simple: the seller can add to M_L an offer y that is sufficiently tempting for the high types in state $\hat{s}$. Since $\hat{s}$ occurs with a positive probability, the consumer will not choose M_L if the self-control cost in the particular state is sufficiently large. To see this more concretely, assume that the low types have no temptation in any state: $V_L^s \sim U_L$ for all s. Assume also that there exists a large enough $t > 0$ such that

$$p_H(\hat{s})U_H(0,t) + (1 - p_H(\hat{s}))U_H(x_L^*) < 0. \tag{11}$$

Since $U_H(x_L^*) > 0$, we have $t > 0$. By Assumption 4, there exists an offer y such that

$$U_H(y) = U_H(0,t),$$
$$U_H(y) + V_H^{\hat{s}}(y) = U_H(x_L^*) + V_H^{\hat{s}}(x_L^*).$$

If we set $M_L \equiv \{x_L^*, y\}$, then

$$W_H^{\hat{s}}(M_L) = U_H(0,t), \text{ and}$$
$$W_H^s(M_L) \le U_H(x_L^*) \quad \text{for all } s \neq \hat{s}.$$

Thus (11) yields $W_H(M_L) < 0$, as desired.[9]

Bounded Quality. An important assumption for the perfect discrimination result is that the optimal decorative offer (e.g., y in Figure 1a) is technologically feasible. Once we relax the assumption and place an upper bound on feasible quality levels q, perfect discrimination may break down. However, the existence of an upper bound on q does not affect the basic form of the optimal schedule: the optimal schedule continues to offer a separate menu for each type and decorate the one intended for the low types.

To see this, suppose that only qualities $q \in [0, \bar{q}]$ are feasible, where $\bar{q} < \infty$. For simplicity, assume that the low types have no temptation. To characterize the optimal schedule, fix a feasible offer x_L such that $U_L(x_L) = 0$. Let $z = (q_z, t_z)$ be the offer that has the highest quality subject to

$$U_H(z) + V_H(z) = U_H(x_L) + V_H(x_L), \quad U_H(z) \ge 0, \quad \text{and } q_z \le \bar{q}.$$

Given this z, let x_H be the offer that maximizes $\pi(x_H)$ subject to $U_H(x_H) = U_H(z)$. Then, given x_L, an optimal schedule is to offer $M_L = \{x_L, z\}$ and $M_H = \{x_H\}$. The seller gains strictly from

[9] The same result extends if stochastic temptation is introduced into the "different model of preferences with temptation" introduced before.

offering two distinct menus and including z in M_L.

An interesting implication of the existence of quality limit is that the seller may prefer a technology with a higher quality limit even if it is technologically inefficient. By raising $\bar{q}$, the seller gets more freedom in the choice of z and thereby can tempt consumers more severely. As an illustration, suppose that the seller has two options for technological investments, A and B. Option A leads to a technology with cost function $C_A(q) = c_A q^2/2$ and the upper bound $\bar{q}_A$. Option B leads to $C_B(q) = c_B q^2/2$ and $\bar{q}_B$, where $c_B > c_A$ and $\bar{q}_B > \bar{q}_A$. Then, although technology A has lower marginal costs, it is possible that technology B yields higher profits than A. For example, suppose $(c_A, \bar{q}_A) = (2, 0.8)$, $(c_B, \bar{q}_B) = (2.2, 2.4)$, $n_L = n_H = 0.5$, and preferences given by

$$U_L(q,t) = V_L(q,t) = q - t, \quad U_H(q,t) = 1.5q - t, \quad \text{and } V_H(q,t) = 1.6q - t.$$

Then the maximum profit under technology A is 0.33 while technology B attains 0.34.

3.3 Downward Temptation

We now turn to the case where the high-type consumers have downward temptation. As mentioned in the introduction, downward temptation is not unusual. In a number of contexts, downward temptation is actually the norm.

If the high types have downward temptation, the seller's problem is considerably different. In particular, the seller cannot achieve perfect discrimination. The reason is that there is no way of decorating $\{x_L^*\}$ that completely eliminates the high types' incentive to mimic the low types. To see this, suppose that perfect discrimination is possible: there exists a feasible schedule of the form (M_L, x_L^*, M_H, x_H^*). Since each consumer's entire ex-ante surplus is extracted, ex-ante IR binds. In particular, $W_H(M_H) = 0$. By ex-ante IC, $W_H(M_L) \le W_H(M_H) = 0$, i.e.,

$$\max_{x \in M_L} \big[U_H(x) + V_H(x)\big] - \max_{x \in M_L} V_H(x) \le 0. \tag{12}$$

Let x^{U+V} and x^V denote solutions to the maximization problems in (12). Then

$$U_H(x^{U+V}) + V_H(x^{U+V}) \le V_H(x^V). \tag{13}$$

Since $x_L^* \in M_L$ and $U_H(x_L^*) + V_H(x_L^*) > 0$,[10] the left-hand side of (13) is strictly positive. Therefore $V_H(x^V) > 0$. Now, a critical observation is that since $V_H \prec U_H$, $V_H(x^V) > 0$ implies $U_H(x^V) > 0$. In words, if a choice gives a positive utility when his valuation is low, then it surely gives a positive value when the valuation is high. Therefore, $V_H(x^V) < U_H(x^V) + V_H(x^V) \le U_H(x^{U+V}) + V_H(x^{U+V})$, which is a contradiction with (13).

To characterize optimal schedules, it is useful to classify the high-type consumers further, based on the intensity of their downward temptation.

Definition. The high-type consumers' downward temptation is ***weak*** if $V_H \succ U_L + V_L$ and ***strong*** if $V_H \prec U_L + V_L$.

Since the high types have downward temptation, their marginal value for quality is lower under

[10] This is because $U_L(x_L^*) + V_L(x_L^*) \ge 0$, $U_H + V_H \succ U_L + V_L$, and $x_L^* \gg 0$.

temptation. If the high types' downward temptation is weak, their marginal value when they are fully tempted is not as low as the marginal value of the low types who are exercising self-control. We first examine the case where the high types have weak temptation.

3.3.1 Weak Downward Temptation

We first show that if the high types have weak downward temptation, the seller does not gain from offering $M_L \supsetneq \{x_L\}$. The only reason for the seller to decorate M_L is to lower $W_H(M_L)$ and weaken the ex-ante IC for the high types, but this is not possible. If the seller offers a "plain" menu $\{x_L\}$ for the low types, the high types' ex-ante utility from the menu is $W_H(\{x_L\}) = U_H(x_L)$. We now show that there exists no menu $M_L \supsetneq \{x_L\}$ such that $W_H(M_L) < U_H(x_L)$. To see this, take any menu $M_L \supsetneq \{x_L\}$ that preserves ex-post IC for the low types. Let $z \in M_L$ be an offer that maximizes V_H. Ex-post IC for the low types and weak downward temptation imply $z \geq x_L$ (the standard monotonicity argument with IC). This and $U_H \succ V_H$ imply $U_H(z) \geq U_H(x_L)$. Therefore

$$W_H(M_L) = \max_{y \in M_L} \Big[U_H(y) + V_H(y) \Big] - V_H(z) \geq U_H(z) \geq U_H(x_L), \tag{14}$$

which proves our claim.

To complete the characterization of optimal schedules, let (M_L, x_L, M_H, x_H) be an optimal schedule. By ex-ante IC, $W_H(M_L) \leq W_H(M_H)$. By $W_H(M_L) = U_H(x_L)$ and (10),

$$U_H(x_L) \leq U_H(x_H) + V_H(x_H) - \max\{0, V_H(x_H)\}. \tag{15}$$

By Lemma 1,

$$F_L(x_L) \geq 0. \tag{16}$$

Therefore, (15) and (16) constitute a necessary condition for optimality. They are also sufficient, as the following proposition shows.

Proposition 2. *Suppose that the high-type consumers have weak downward temptation. Then, for any optimal schedule (M_L, x_L, M_H, x_H), the pair (x_L, x_H) is a solution of maximizing $n_L\pi(x_L) + n_H\pi(x_H)$ subject to* (15) *and* (16)*. Conversely, if a pair (x_L, x_H) solves the constrained maximization, then $(\{x_L\}, x_L, \{x_H\}, x_H)$ is an optimal schedule,* (15) *and* (16) *hold with equality, and $F_L(x_H) < 0$.*

This result allows us to describe the optimal schedule graphically. See Figure 2. Since $F_L(x_L) = 0$ at the optimum, one option for the seller is to set $x_L = x_L^*$. With this choice, the set of feasible choices of x_H is given by the kinked curve that follows the indifference curve of U_H from x_L^* to y and then follows the indifference curve of $U_H + V_H$ to the right. This kinked curve is the set of offers x_H that satisfy (15) with equality. Let x_H denote the offer that is most profitable on the kinked curve. To consider the interesting case, suppose, as in the figure, that this offer x_H lies on the indifference curve of U_H.

We now decrease the quality level offered to the low types along $F_L = 0$. Then, the kinked curve shifts upwards and therefore the most profitable offer that can be given to the high types moves up (straight if preferences are quasi-linear). However, it eventually hits the $V_H = 0$ curve. At this point, the offer for the high types is at the kink and remains so for a while as we continue

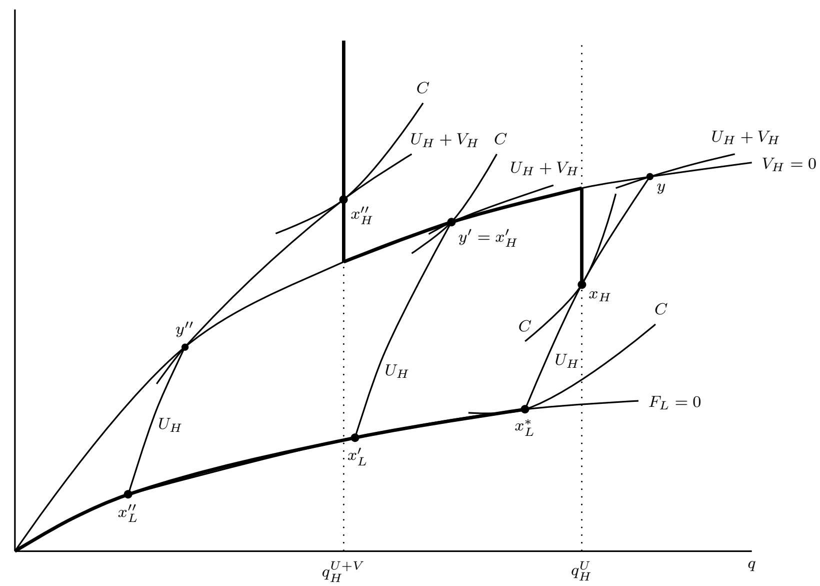


Figure 2: Optimal schedule when the high types have weak downward temptation

moving x_L. For example, if x'_L is the offer to the low types, the most profitable offer that can be given to the high types is y'. As we continue, the offer to the high types eventually moves on to the indifference curve of $U_H + V_H$, as depicted by x''_H. The pair of thick lines in the figure show the paths of the offers. Then the seller's problem is to choose an offer $\hat{x}_L$ between the origin and x^*_L on $F_L = 0$, to maximize the profits $n_L \pi(\hat{x}_L) + n_H \pi(\hat{x}_H)$, where $\hat{x}_H$ is the associated offer for the high types. Then $(\{\hat{x}_L\}, \hat{x}_L, \{\hat{x}_H\}, \hat{x}_H)$ is an optimal schedule.

The figure also illustrates why the seller offers multiple menus. If the seller offers a single menu, then by ex-post IC, x_H has to be on the indifference curve of $U_H + V_H$ that passes through x_L. Since the indifference curves of $U_H + V_H$ are flatter than those of U_H, one can see from the figure that by offering two menus, the seller expands the set of offers that can be assigned to the high types for any given $x_L \neq 0$ on $F_L = 0$.

Important properties of optimal schedules are summarized as follows.

Proposition 3. *If the high-type consumers have weak downward temptation, any optimal schedule satisfies the following properties:*

(i) The low types get a zero ex-ante surplus: $W_L(M_L) = 0$.

(ii) The high types get a strictly positive ex-ante surplus (i.e., information rent): if $x_L \neq 0$, *then* $W_H(M_H) > 0$.

(iii) The offer x_H *is generally not socially efficient in the sense that, at* x_H, *the iso-profit curve is not necessarily tangent with either the indifference curve of* U_H *or that of* $U_H + V_H$.

(iv) The seller offers different menus to different types: if $x_L \neq 0$, *then* $M_L \neq M_H$.

An implication of the characterization is that, in contrast with the upward temptation case, the standard nonlinear pricing theory is robust to the introduction of downward temptation. First, as in the standard case, the seller has no incentive to decorate menus. Second, the profile of offers that the consumers choose, i.e., (x_L, x_H), is close to that in the standard problem if the preferences are close to being standard. Indeed, (15) and (16) can be rewritten as

$$U_H(x_L) \leq \min\{U_H(x_H) + V_H(x_H), U_H(x_H)\},$$
$$0 \leq \min\{U_L(x_L) + V_L(x_L), U_L(x_L)\}.$$

As V_γ converges to U_γ, the pair of inequalities converges to the critical pair of incentive constraints in the standard problem with utility functions U_γ.

3.3.2 Strong Downward Temptation

We now examine the case where the high types have strong downward temptation: $V_H \prec U_L + V_L$. Since $V_L \prec V_H$, we have $V_L \prec U_L + V_L$, i.e., the low types also have downward temptation.

If the high types have strong downward temptation, the seller can bunch all the consumers into a single menu without any loss. To see this, suppose, for the moment, that ex-post IR binds for the low types:

$$U_L(x_L) + V_L(x_L) = 0. \tag{17}$$

Since $V_H \prec U_L + V_L$, we obtain $V_H(x_L) \leq 0$, i.e., x_L is not as tempting as $(0,0)$ for the low types. Furthermore, ex-post IC for the low types implies that no other offer in M_L is more tempting than $(0,0)$ and thus $\max_{x \in M_L} V_H(x) = 0$. This implies

$$W_H(M_L) \geq U_H(x_L) + V_H(x_L). \tag{18}$$

Since $\max_{x \in M_H} V_H(x) \geq 0$, we have $W_H(M_H) \leq U_H(x_H) + V_H(x_H)$. This, together with (18) and ex-ante IC, implies

$$U_H(x_H) + V_H(x_H) \geq U_H(x_L) + V_H(x_L).$$

The inequality implies that even if x_H and x_L are offered in the same menu, the high types continue to have an incentive to choose x_H. This and (17) imply that $\{x_L, x_H\}$ satisfies the critical pair of incentive constraints in the standard problem with utility functions $U_\gamma + V_\gamma$, suggesting that the optimal schedule in the standard problem is also optimal here. However, to establish this claim, there are two issues to be resolved.

First, the above argument starts by assuming (17). What if the seller sets x_L such that $U_L(x_L) + V_L(x_L) > 0$? The advantage of doing so is that it enables the seller to decorate M_L with a low quality good and reduce the high types' ex-ante utility for M_L. The most effective decoration is $(\hat{q}, 0)$ defined by $U_L(\hat{q}, 0) + V_L(\hat{q}, 0) = U_L(x_L) + V_L(x_L)$. The offer $(\hat{q}, 0)$ is the most tempting offer that the seller can include in M_L without disturbing ex-post IC for the low types. This offer gives $V_H(\hat{q}, 0) > 0$, and thus lowers the high types' ex-ante utility from M_L and enables the seller to charge more for x_H in a separate menu. Therefore, by choosing x_L for which ex-post IR does not bind, the seller earns less from the low types but can earn more from the high types. We next show that despite being feasible, this decoration strategy does not increase the overall profit if preferences are quasi-linear. That is, with the restricted domain of preferences, (17) is satisfied at any optimal schedule.

Lemma 2. *Suppose that the high-type consumers have strong downward temptation and the or-*

dinal preferences induced by $U_\gamma + V_\gamma$ *and* V_γ *are quasi-linear for each* γ.[11] *Then, for any optimal schedule,* $U_L(x_L) + V_L(x_L) = 0$.

With this result and the restricted domain of preferences, the previous argument implies that for any optimal schedule (M_L, x_L, M_H, x_H), the simplified menu $\{x_L, x_H\}$ satisfies all of the ex-post incentive constraints. Therefore, to conclude that offering a single menu $\{x_L, x_H\}$ is optimal, it remains to show that the menu satisfies ex-ante IR for each type. To this end, we show that ex-ante IR is vacuous under downward temptation.

Lemma 3. *For consumers with downward temptation, any menu satisfies ex-ante IR.*

This yields the following characterization of optimal schedules.

Proposition 4. *Suppose that the high-type consumers have strong downward temptation and the ordinal preferences of* $U_\gamma + V_\gamma$ *and* V_γ *are quasi-linear for each* γ. *Then, for any optimal schedule* (M_L, x_L, M_H, x_H), *the pair* (x_L, x_H) *is a solution to the standard problem with utility functions* $U_\gamma + V_\gamma$. *Conversely, if* (x_L, x_H) *is a solution to the standard problem with utility functions* $U_\gamma + V_\gamma$, $(\{x_L, x_H\}, x_L, \{x_L, x_H\}, x_H)$ *is an optimal schedule.*

This result implies that the standard theory of nonlinear pricing is robust to the introduction of strong downward temptation: all that the analyst has to do is use the consumers' utility functions under self-control ($U_\gamma + V_\gamma$). The qualitative properties of the optimal schedule are thus unchanged.

4 Entry Fees

We have so far assumed that all menus contain $(0,0)$, i.e., consumers who do not buy any good do not have to pay. While this is a reasonable assumption for retail stores and restaurants, a number of services charge fees that are independent of service usage (e.g., membership fees). In this section, we extend our analysis to the case where the seller can charge such fees.

In terms of formal modeling, allowing for entry fees is equivalent to removing the assumption that each menu contains $(0,0)$. Thus, a ***menu*** is now a non-empty compact subset $M \subseteq \mathbb{R}^2_+$.

Removing $(0,0)$ from a menu can raise profits in two ways. First, by removing $(0,0)$, the seller can eliminate ex-post IR from the incentive constraints. Note, however, that ex-ante IR remains a constraint, since consumers continue to have the option of not choosing any of the menus offered by the seller. Second, the seller may gain from deleting $(0,0)$ since doing so may also reduce self-control costs for consumers who are tempted by $(0,0)$. Since this raises the consumers' ex-ante utility, the seller can charge more.

As in the standard analysis, schedules list only the offers that consumers can choose and do not necessarily specify the price for all qualities q. If the price for a quality q is not specified, the implicit assumption is that either the seller does not offer the quality, or the price for the quality is prohibitive. For this reason, the entry fee, defined as the price for $q = 0$, may not appear explicitly in the characterization of optimal schedules. Entry fees can be made explicit since any high enough entry fee will preserve all the incentives. To gain more insight, we will characterize the minimum entry fees that support the optimal schedule (Section 4.5). As we will show, the minimum entry

[11] For the ordinal definition of quasi-linearity, see, e.g., Mas-Colell, Whinston, and Green (1995).

fees are sometimes positive, while they are always zero in the standard problem.

4.1 Complete Information

We again start with the complete information case. Since ex-post IR is no longer a constraint, x^*_γ is the most profitable offer subject only to the constraint that $\{x^*_\gamma\}$ satisfies ex-ante IR. Since $\{x^*_\gamma\}$ is truly a singleton and does not induce any temptation, $W_\gamma(\{x^*_\gamma\}) = U_\gamma(x^*_\gamma)$. Thus x^*_γ is simply the offer that maximizes $\pi(x)$ subject to $U_\gamma(x) = 0$. Therefore, the feasibility of entry fees changes x^*_γ only for consumers with downward temptation. Indeed, for these consumers, ex-post IR is a binding constraint when entry fees are not feasible.

4.2 Upward Temptation

If the high types have upward temptation, perfect discrimination remains possible. The construction of the optimal schedule is identical, except that, as just discussed, the feasibility of entry fees changes x^*_L if the low types have downward temptation. The proof is thus omitted.

Proposition 5. *If entry fees are feasible and the high-type consumers have upward temptation, there exists a feasible schedule* (M_L, x_L, M_H, x_H) *such that* $x_L = x^*_L$ *and* $x_H = x^*_H$.

4.3 Weak Downward Temptation

If the high types have weak downward temptation, the feasibility of entry fees changes the basic form of the optimal schedule. To see why, recall that, without entry fees the seller offers two menus and separates the types, and ex-post IR binds for the low types if they have downward temptation. If entry fees are feasible, on the other hand, ex-post IR does not have to be satisfied.

Figure 2 shows another reason why entry fees can increase profits. Without entry fees, x_H is on the indifference curve of either $U_H + V_H$ or U_H. If x_H is on the $U_H + V_H$ curve, the high types are tempted by $(0,0)$ and incur a positive self-control cost. If entry fees are feasible, the seller can remove $(0,0)$ from the menu and raise the high types' ex-ante utility.

To characterize optimal schedules, let (M_L, x_L, M_H, x_H) be any optimal schedule. The proof for (14) remains valid and therefore the seller can set $M_L = \{x_L\}$ without any loss. Hence $W_H(M_L) = U_H(x_L)$. By ex-ante IC, $W_H(M_H) \geq W_H(M_L)$. Since the self-control cost is non-negative, $W_H(M_H) \leq U_H(x_H)$. These inequalities together imply

$$U_H(x_H) \geq U_H(x_L). \tag{19}$$

For the low types, non-negative self-control cost implies $W_L(M_L) \leq U_L(x_L)$ and ex-ante IR implies $W_L(M_L) \geq 0$, yielding

$$U_L(x_L) \geq 0. \tag{20}$$

Inequalities (19) and (20) are nothing but the pair of critical incentive constraints in the standard problem with utility functions U_γ. That is to say, the standard problem with utility functions U_γ faces fewer constraints.

Conversely, let $\{x_H, x_L\}$ denote an optimal menu in the standard problem with utility functions U_γ (possibly $x_L = 0$). We claim that $(\{x_L\}, x_L, \{x_H\}, x_H)$ is an optimal schedule. Indeed, since these menus do not induce any temptation, consumers evaluate them with U_γ. Therefore

$$W_H(\{x_H\}) = U_H(x_H) = U_H(x_L) = W_H(\{x_L\}) \geq 0,$$
$$W_L(\{x_L\}) = U_L(x_L) = 0 \geq U_L(x_H) = W_L(\{x_H\}).$$

This shows that the proposed schedule satisfies ex-ante IR and IC. Since the menus are singletons, ex-post IC holds vacuously. Since (x_L, x_H) maximizes profits subject to (19) and (20), there is no schedule that generates more profits. Thus we have shown

Proposition 6. *Suppose that entry fees are feasible and the high-type consumers have weak downward temptation. Then for any optimal schedule (M_L, x_L, M_H, x_H), the pair (x_L, x_H) solves the standard problem with utility functions U_γ. Conversely, if a pair (x_L, x_H) solves the standard problem with utility functions U_γ, then $(\{x_L\}, x_L, \{x_H\}, x_H)$ is an optimal schedule.*

By comparing this result with Figure 2, we can see how entry fees affect the optimal schedule. First, if the low types have downward temptation, then $U_L \succ F_L$ and thus the feasibility of entry fees moves up the curve on which x_L has to be located. In this case, therefore, M_L charges a positive entry fee. Note that if $x_L \neq 0$, then $U_L(x_L) + V_L(x_L) < 0$. Therefore, ex post, a low-type consumer prefers to buy nothing, but $(0,0)$ is no longer an option because he has committed to an entry fee. Having made the commitment, he finds x_L to be an optimal choice.

As Figure 2 shows, another effect of the feasibility of entry fees is that the set of offers x_H that can be assigned to the high types faces no constraint from the indifference curves of $U_H + V_H$. Since $U_H \succ U_H + V_H$, this effect is also favorable to the seller. If the optimal x_H falls in the expanded part of the constraint set, M_H also charges an entry fee.

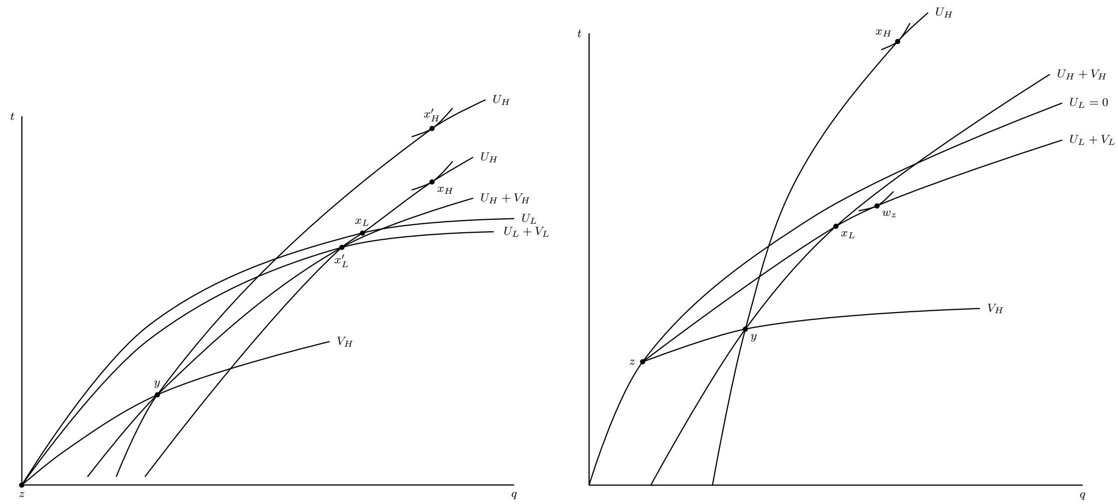
4.4 Strong Downward Temptation

If the high types' downward temptation is strong, the optimal schedule differs from that in the case of weak downward temptation. The firm now gains from decorating M_L. To see why, let $(\{x_L\}, x_L, \{x_H\}, x_H)$ be the optimal schedule in Proposition 6. Suppose further that the scale of the function V_L is small and therefore $U_L + V_L$ and U_L induce similar indifference curves (i.e., the low types have almost infinite willpower). The seller can then offer a pair of menus given by $M'_L = \{x'_L, z = (0,0)\}$ and $M'_H = \{x'_H\}$ depicted in Figure 3a. Then

$$W_H(M'_L) = U_H(y) + V_H(y) - V_H(y) = U_H(y) < U_H(x_L) = W_H(\{x_L\}).$$

The inequality implies that the presence of $(0,0)$ in M'_L decreases the high types' ex-ante utility from the menu for the low types. This allows the seller to extract more surplus from the high types. In the figure, x'_H is indeed located above x_H. The difference between x'_H and x_H is bounded away from zero as x'_L converges to x_L.

An optimal schedule can be identified as follows. See Figure 3b. Pick an offer $z = (q_z, t_z)$ such that either $U_L(z) = 0$ or $t_z = 0$. Then pick another offer x_L that is indifferent to z for $U_L + V_L$ and such that $x_L \geq z$ (so that $U_L(x_L) \geq 0$). Given a choice of z and x_L, compute the high types' ex-ante utility from $\{x_L, z\}$, which equals $U_H(y)$. Then, among the offers x_H such that $U_H(x_H) = U_H(y)$, choose one that maximizes $\pi(x_H)$. Then let $\Pi(z, x_L) \equiv n_H \pi(x_H) + n_L \pi(x_L)$ denote the expected



(a) Decoration is profitable **(b)** Optimal schedule

Figure 3: Entry fees are feasible and the high types have strong downward temptation

profit given (z, x_L). To identify an optimal schedule, first choose x_L between z and w_z on the $U_L + V_L$ indifference curve, to maximize $\Pi(z, x_L)$. This gives an optimal offer $x_L(z)$ as a function of z. Finally, choose z to maximize $\Pi(z, x_L(z))$. Given a maximizer z, let (x_L, x_H) be the associated pair of offers. Then $(\{z, x_L\}, x_L, \{x_H\}, x_H)$ is an optimal schedule.

Formally, let Z be the set of offers $z = (q_z, t_z)$ such that either $[U_L(z) = 0$ and $0 \le z \le x_L^*]$ or $t_z = 0$. For any $z \in Z$, let w_z be the offer that maximizes $\pi(w_z)$ subject to $U_L(w_z) + V_L(w_z) = U_L(z) + V_L(z)$ and $w_z \ge z$. For any $z \in Z$, let $X_L(z)$ be the set of offers x_L such that $U_L(x_L) + V_L(x_L) = U_L(z) + V_L(z)$ and $z \le x_L \le w_z$. For any $z \in Z$ and $x_L \in X_L(z)$, let $X_H(z, x_L)$ denote the offer x_H that maximizes $\pi(x_H)$ subject to $U_H(x_H) = U_H(x_L) + V_H(x_L) - V_H(z)$. Then

Proposition 7. *Suppose that entry fees are feasible and the high-type consumers have strong downward temptation. Let (z, x_L) be a pair that maximizes $n_L \pi(x_L) + n_H \pi(X_H(z, x_L))$ subject to $z \in Z$ and $x_L \in X_L(z)$. Then $(\{z, x_L\}, x_L, \{x_H\}, x_H)$ with $x_H = X_H(z, x_L)$ is an optimal schedule.*[12]

4.5 Supporting Entry Fees

In this section, we explicitly compute the entry fees that appear only implicitly in the characterization of optimal schedules. That is, given an optimal schedule, we identify the entry fee level for each menu so that the incentive constraints are preserved. We say that a pair of entry fees $(e_L, e_H) \in \mathbb{R}^2_+$ *supports* the optimal schedule if adding the offers $(0, e_L)$ and $(0, e_H)$ to M_L and M_H, respectively, preserves the feasibility of the schedule. Here, e_γ is the fee that consumers are asked to pay if they choose M_γ and end up buying nothing. Since any pair of sufficiently high entry fees preserve feasibility, we look for the "minimal" such pairs.

Formally, for a given feasible schedule (M_L, x_L, M_H, x_H), a ***supporting entry-fee profile*** is a pair $(e_L, e_H) \in \mathbb{R}^2_+$ such that $(M_L \cup \{(0, e_L)\}, x_L, M_H \cup \{(0, e_H)\}, x_H)$ is feasible. It can be shown

[12]The proposition allows for z such that $q_z > 0$ and $t_z = 0$, in which case $U_L(z) + V_L(z) > 0$. However, this case can be ruled out if preferences are quasi-linear. Since this can be proved with a straightforward modification of the proof of Lemma 2, the proof is omitted.

that for any feasible schedule, there exists a unique ***minimum supporting entry-fee profile*** $e^* = (e^*_L, e^*_H)$, which is the supporting entry-fee profile such that for any other supporting entry-fee profile e, we have $e \geq e^*$. See Appendix A.7 for the proof of the uniqueness.

Adding $(0, e_\gamma)$ to M_γ affects the incentive constraints in three ways. First, if e_γ is too low, type γ may prefer $(0, e_\gamma)$ to x_γ ex post: ex-post IC for γ may be disturbed. Second, if e_γ is too low, $(0, e_\gamma)$ may be more tempting than other items in M_γ for γ. Then the type's self-control cost goes up, which may induce the type to switch to another menu: the ex-ante IC for γ may be disturbed. Third, if e_γ is too low, the other type ($\gamma' \neq \gamma$) may find $(0, e_\gamma)$ better than the other items in M_γ. This may increase the type's ex-ante utility from M_γ and induce the type to choose M_γ over $M_{\gamma'}$: ex-ante IC for γ' may be disturbed.

Proposition 8. *For any optimal schedule (M_L, x_L, M_H, x_H) in Propositions 5–7, the minimum supporting entry-fee profile is $(\hat{e}_L, \hat{e}_H)$ defined by*

$$V_\gamma(0, \hat{e}_\gamma) = \min\{0, \max_{x \in M_\gamma} V_\gamma(x)\}, \qquad \gamma = L, H.$$

In words, if type γ's most tempting offer in M_γ is more tempting than the option of no purchase and no payment ($\max V_\gamma \geq 0$), then $\hat{e}_\gamma = 0$. If the option of no purchase and no payment is even more tempting than the most tempting offer in M_γ ($\max V_\gamma \leq 0$), then $\hat{e}_\gamma$ is such that $(0, \hat{e}_\gamma)$ is exactly as tempting as the most tempting offer.

Notice that $\hat{e}_\gamma$ is independent of U_γ and the other type's preferences. This implies that, among the three effects discussed above, only the second one is binding: $\hat{e}_\gamma$ is determined solely by the constraint of not affecting γ's most tempting choice. Intuitively, the reason is as follows. First, if type γ has upward temptation, ex-post IC is not a binding constraint for $\hat{e}_\gamma$ since an offer of the form $(0, e_\gamma)$ is not a serious choice in the ex-post self-controlled decision. Since the untempted part prefers x_γ to $(0, 0)$ (otherwise the type would not choose the menu), no amount of upward temptation makes $(0, 0)$, let alone $(0, e_\gamma)$, an appealing choice. Second, if the consumer has downward temptation, an offer of the form $(0, e_\gamma)$ is more important when the consumer is fully tempted (i.e., when he maximizes V_γ) than when the consumer is exercising self-control (when he maximizes $U_\gamma + V_\gamma$). This allows us to ignore the effect on the optimal choice in the $U_\gamma + V_\gamma$ problem. Finally, to see that ex-ante IC for the other type ($\gamma' \neq \gamma$) is not a binding constraint for $\hat{e}_\gamma$, note that if $(0, \hat{e}_\gamma)$ does increase the ex-ante utility of γ' in M_γ, then $(0, \hat{e}_\gamma)$ has to be the ex-post optimal choice for γ' in M_γ. But, if the outcome of choosing the menu M_γ is to buy nothing and pay a non-negative entry fee, the consumer prefers not choosing the menu. Thus the constraint of not inducing γ' to enter M_γ is not binding for $\hat{e}_\gamma$.

While Proposition 8 gives the exact level of the minimum supporting entry fee for each menu, one may be particularly interested in whether entry fees are positive at the optimal schedule. As Table 1 summarizes, Proposition 8 has a few implications: (i) for each type, $\hat{e}_\gamma = 0$ if γ has upward temptation, (ii) $\hat{e}_L > 0$ if the low types have downward temptation and the high types do not have strong downward temptation, and (iii) $\hat{e}_H > 0$ if the high types have strong downward temptation.

Among these implications, (i) might be particularly interesting since it says that if $\hat{e}_\gamma > 0$, type γ has downward temptation. Thus, if the data for a particular service category show that positive entry fees are charged consistently in menus targeted for a certain segment of consumers, this may be an indication of $\hat{e}_\gamma > 0$ for these consumers. Our theory then implies that these

	High Type's Temptation		
Low Type's Temptation	Upward	Weak Downward	Strong Downward
Upward	$\hat{e}_L = 0$, $\hat{e}_H = 0$	$\hat{e}_L = 0$, $\hat{e}_H \geq 0$	
Downward	$\hat{e}_L > 0$, $\hat{e}_H = 0$	$\hat{e}_L > 0$, $\hat{e}_H \geq 0$	$\hat{e}_L \geq 0$, $\hat{e}_H > 0$

Table 1: Signs of minimum supporting entry fees at the optimal schedule

consumers have downward temptation.

It is thus worth noting that implication (i) discussed above extends to all feasible schedules.

Proposition 9. *Let (M_L, x_L, M_H, x_H) be any feasible schedule and (e^*_L, e^*_H) be the minimum supporting entry-fee profile for the schedule. Then for any type γ, if $e^*_\gamma > 0$, then type γ has downward temptation.*

5 Concluding Remarks

Consumers' temptation and self-control appear extremely relevant for firms' pricing decisions. By using a recent advance in decision theory together with the classical framework of nonlinear pricing, we provide a formal theoretical analysis of a firm's optimal response to consumers with temptation. Because of the consumers' behavioral features, the number of menus and entry fees matter for the seller's problem and thus our theory provides a link between these decisions of the seller and the demand side of the market. Since the standard nonlinear pricing problem is subsumed as a special case, we were also able to clarify whether the standard theory is robust to the introduction of the behavioral features.

Probably, the most restrictive assumption in our analysis is that there are only two types of consumers. While this assumption certainly provides a simplest framework to identify new features of the optimal pricing, it may miss features that emerge only in a more general setting. As we showed in Appendix, the perfect discrimination result does extend to many types. However, we do not offer characterizations of optimal schedules when perfect discrimination is not possible. Once we allow for more types, the complexity of the analysis multiplies rapidly since there are a large number of menu configurations that need to be considered.

Another unrealistic assumption in our analysis is that consumers have complete information about the menus offered by the seller and their own preferences. In reality, we often have to visit a store to find out what is offered and what we are looking for. This gives an advantage to sellers with large selections and makes advertisements important.

Acknowledgments

We are grateful to Larry Samuelson for posing a question that led us to start this paper. We are also grateful to Masahiro Ashiya, Eric Bond, Faruk Gul, Noritsugu Nakanishi, Ran Spiegler, and Manabu Toda for helpful comments and especially to Matthew Shum for his input at the initial stage of the research.

A Appendix

A.1 Proof of Proposition 1

Let y be the intersection of the $U_H = 0$ curve and the indifference curve of $U_H + V_H$ through x_L^*:

$$\begin{aligned} U_H(y) + V_H(y) &= U_H(x_L^*) + V_H(x_L^*), \qquad (21) \\ U_H(y) &= 0. \end{aligned}$$

The offer y is well defined by Assumption 4. Since $F_L(x_L^*) = 0$, $x_L^* \gg 0$, and $F_L \prec U_H \prec U_H + V_H$, we have $y \gg x_L^*$.

If $V_L(y) \leq V_L(x_L^*)$ (which happens if $V_L \prec U_H + V_H$), let $z = y$. Otherwise, let z be the intersection of the indifference curve of V_L through x_L^* and the indifference curve of V_H through y:

$$\begin{aligned} V_L(z) &= V_L(x_L^*), \\ V_H(z) &= V_H(y). \qquad (22) \end{aligned}$$

We claim that a schedule $(\{x_L^*, z\}, x_L^*, \{x_H^*\}, x_H^*)$ is feasible and thus optimal. By the definition of perfect discrimination offers, ex-post IR is satisfied for each type and $\{x_H^*\}$ satisfies ex-ante IR. Since $\{x_H^*\}$ contains only x_H^* and $(0,0)$, ex-post IC is vacuous for the high types. Thus, it remains to prove ex-ante IC for each type and ex-ante IR and ex-post IC for the low types.

Ex-ante IC for H: We first show that the high types' ex-post optimal choice from $\{x_L^*, z\}$ is x_L^*. Indeed, $F_L(x_L^*) = 0$ and $F_L \prec U_H + V_H$ imply

$$U_H(x_L^*) + V_H(x_L^*) \geq 0. \qquad (23)$$

Further, $z \geq y$, $V_H(z) = V_H(y)$, and $V_H \succ U_H + V_H$ imply

$$U_H(z) + V_H(z) \leq U_H(y) + V_H(y) = U_H(x_L^*) + V_H(x_L^*). \qquad (24)$$

This and (23) imply that x_L^* is an optimal choice from $\{x_L^*, z\}$ for $U_H + V_H$.

We show next that the most tempting choice in $\{x_L^*, z\}$ is z. Indeed, we have $V_H(y) \geq V_H(x_L^*)$ by (21), $y \gg x_L^*$, and $V_H \succ U_H + V_H$. Thus by (22), $V_H(z) \geq V_H(x_L^*)$. Further, $F_L(x_L^*) = 0$ and $F_L \prec V_H$ imply $V_H(x_L^*) \geq 0$ and hence $V_H(z) \geq 0$.

These facts together imply

$$W_H(\{x_L^*, z\}) = U_H(x_L^*) + V_H(x_L^*) - V_H(z) = U_H(y) + V_H(y) - V_H(y) = 0 = W_H(\{x_H^*\}).$$

Ex-post IC for L: By $z \geq x_L^*$, (24), and $U_H + V_H \succ U_L + V_L$, we have $U_L(x_L^*) + V_L(x_L^*) \geq U_L(z) +$

$V_L(z)$.

Ex-ante IR for L: By the construction of z, $V_L(x_L^*) \geq V_L(z)$. This and ex-post IC imply that the inclusion of z does not affect the low types' ex-ante utility: $W_L(\{x_L^*, z\}) = W_L(\{x_L^*\})$. Since x_L^* is a perfect discrimination offer, $W_L(\{x_L^*\}) = 0$.

Ex-ante IC for L: Suppose, by contradiction, that $W_L(\{x_H^*\}) > 0$. This is possible only if $U_L(x_H^*) + V_L(x_H^*) > 0$, which implies

$$W_L(\{x_H^*\}) = U_L(x_H^*) + V_L(x_H^*) - \max\{0, V_L(x_H^*)\}. \tag{25}$$

But since $x_H^* \gg 0$ and $U_H \succ F_L$, we have $F_L(x_H^*) < 0$, which implies that (25) is negative, a contradiction. This proves $W_L(\{x_H^*\}) \leq 0$, as desired. Q.E.D.

A.2 Proof of Proposition 2

The proof consists of a few lemmas.

Lemma 4. *If a schedule (M_L, x_L, M_H, x_H) is feasible, (x_L, x_H) satisfies*

$$F_L(x_L) \geq 0, \tag{26}$$

$$U_H(x_L) \leq U_H(x_H) + V_H(x_H) - \max\{0, V_H(x_H)\}. \tag{27}$$

Proof. See the main text.

Lemma 5. *If a pair (x_L, x_H) satisfies* (26), (27), *and $F_L(x_H) < 0$, then $(\{x_L\}, x_L, \{x_H\}, x_H)$ is a feasible schedule.*

Proof. We start with the low type. Since $F_L(x_L) \geq 0$, ex-post IR and ex-ante IR follow from Lemma 1. Thus, it remains to prove ex-ante IC. For this, it suffices to prove $W_L(\{x_H\}) \leq 0$. So, suppose $W_L(\{x_H\}) > 0$. This is possible only if $U_L(x_H) + V_L(x_H) > 0$. Thus $W_L(\{x_H\}) = U_L(x_H) + V_L(x_H) - \max\{0, V_L(x_H)\}$, which is negative since $F_L(x_H) < 0$, a contradiction.

Now, consider the high type. Since $F_L(x_L) \geq 0$ and $F_L \prec U_H$, we obtain $U_H(x_L) \geq 0$. Thus, the right-hand side of (27) is non-negative. This implies $U_H(x_H) + V_H(x_H) \geq 0$, so ex-post IR is satisfied. Thus

$$W_H(\{x_H\}) = U_H(x_H) + V_H(x_H) - \max\{0, V_H(x_H)\} \geq U_H(x_L) \geq 0,$$

which proves ex-ante IR. In the main text, we proved $W_H(\{x_L\}) = U_H(x_L)$. Thus $W_H(\{x_L\}) \leq W_H(\{x_H\})$, which proves ex-ante IC. Q.E.D.

Lemma 6. *If a pair (x_L, x_H) satisfies* (26), (27), *and $F_L(x_H) \geq 0$, then there exists a pair (x'_L, x'_H) that satisfies* (26), (27), *$F_L(x'_H) < 0$, and $n_L\pi(x'_L) + n_H\pi(x'_H) > n_L\pi(x_L) + n_H\pi(x_H)$.*

Proof. Let (x_L, x_H) be as in the lemma. Then $F_L(x_H) \geq 0$ and $F_L(x_L) \geq 0$. Since x_L^* is the most profitable offer satisfying $F_L \geq 0$,

$$\pi(x_L^*) \geq \max\{\pi(x_L), \pi(x_H)\}. \tag{28}$$

Since the curve $F_L = 0$ is tangent with the (differentiable) iso-profit curve at x_L^* and $V_H \succ F_L$, there exists an offer $x'_H \gg x_L^*$ such that

$$\begin{aligned} V_H(x'_H) &> V_H(x_L^*), \\ \pi(x'_H) &> \pi(x_L^*). \end{aligned} \tag{29}$$

We claim that (x_L^*, x'_H) is a desired pair. Since $U_H \succ V_H$, $U_H(x'_H) > U_H(x_L^*)$. Since $V_H \succ F_L$ and $F_L(x_L^*) = 0$, we have $V_H(x_L^*) \geq 0$, and hence $V_H(x'_H) > 0$. Thus

$$U_H(x'_H) + V_H(x'_H) - \max\{0, V_H(x'_H)\} = U_H(x'_H) > U_H(x_L^*).$$

This implies that (x_L^*, x'_H) satisfies (27). Since x_L^* maximizes π subject to $F_L \geq 0$, (29) implies $F_L(x'_H) < 0$. By (29) and (28), (x_L^*, x'_H) generates larger profits than (x_L, x_H). Q.E.D.

Lemma 7. *For any optimal schedule (M_L, x_L, M_H, x_H), the pair (x_L, x_H) maximizes $n_L\pi(x_L) + n_H\pi(x_H)$ subject to* (26) *and* (27)*.*

Proof. If not, there exists a pair (x'_L, x'_H) that dominates (x_L, x_H). Lemma 6 implies that, among such pairs, there exists one that also satisfies $F_L(x'_H) < 0$. Then by Lemma 5, $(\{x'_L\}, x'_L, \{x'_H\}, x'_H)$ is feasible. But this schedule generates more profits than (M_L, x_L, M_H, x_H), a contradiction. Q.E.D.

Lemma 8. *If a pair (x_L, x_H) maximizes $n_L\pi(x_L) + n_H\pi(x_H)$ subject to* (26) *and* (27)*, then $(\{x_L\}, x_L, \{x_H\}, x_H)$ is an optimal schedule.*

Proof. By Lemma 6, $F_L(x_H) < 0$. Thus by Lemma 5, $(\{x_L\}, x_L, \{x_H\}, x_H)$ is a feasible schedule. Suppose that this schedule is not optimal. Then there exists a feasible schedule (M'_L, x'_L, M'_H, x'_H) that dominates it. But by Lemma 4, (x'_L, x'_H) satisfies (26) and (27). Since (x_L, x_H) maximizes profits subject to (26) and (27), we obtained a contradiction. Q.E.D.

Lemma 9. *If a pair (x_L, x_H) maximizes $n_L\pi(x_L) + n_H\pi(x_H)$ subject to* (26) *and* (27)*, then* (26) *and* (27) *hold with equality and $F_L(x_H) < 0$.*

Proof. If (27) is not binding, x_H can be replaced with $x_H + (0, \varepsilon)$ for a small $\varepsilon > 0$. If (26) is not binding, x_L can be replaced with $x_L + (0, \varepsilon)$. Lemma 6 implies $F_L(x_H) < 0$. Q.E.D.

A.3 Proof of Proposition 3

(i) Since $F_L(x_L) = 0$, Lemma 1 implies $W_L(M_L) = 0$.

(ii) Since (15) binds, $W_H(M_H) = U_H(x_L)$. Since $F_L(x_L) = 0$ and $F_L \prec U_H$, we have $U_H(x_L) > 0$ if $x_L \neq 0$.

(iii) In Figure 2, if the optimal schedule offers a pair like $\{x'_L\}$ and $\{x'_H\}$, then at x'_H, the iso-profit curve is not tangent with the indifference curve of either U_H or $U_H + V_H$.[13]

(iv) Suppose, by contradiction, that $x_L \neq 0$ and $M_L = M_H$. There are two possible cases. First, suppose $V_H(x_H) \geq 0$. Then (15) with equality implies $U_H(x_L) = U_H(x_H)$. Since $F_L(x_H) < 0 =$

[13] One can easily find a numerical example where the optimal schedule offers a pair like (x'_L, x'_H) in the figure.

$F_L(x_L)$ and $U_H \succ F_L$, we have $x_H \gg x_L$. Since $U_H + V_H \prec U_H$, we obtain $U_H(x_L) + V_H(x_L) > U_H(x_H) + V_H(x_H)$, a contradiction with ex-post IC. Next, consider the case where $V_H(x_H) \leq 0$. Then (15) with equality implies $U_H(x_L) = U_H(x_H) + V_H(x_H)$. Since $F_L(x_L) = 0$, $x_L \neq 0$, and $V_H \succ F_L$, we obtain $V_H(x_L) > 0$. Thus $U_H(x_L) + V_H(x_L) > U_H(x_L) = U_H(x_H) + V_H(x_H)$, which is again a contradiction with ex-post IC. Q.E.D.

A.4 Proof of Lemma 2

It suffices to prove the following.

Lemma 10. *Suppose that the high-type consumers have strong downward temptation and the ordinal preferences of $U_\gamma + V_\gamma$ and V_γ are quasi-linear for each γ. Then, for any feasible schedule (M_L, x_L, M_H, x_H) such that $U_L(x_L) + V_L(x_L) > 0$, there exists a feasible schedule (M'_L, x'_L, M'_H, x'_H) such that*

$$\begin{aligned} n_L\pi(x'_L) + n_H\pi(x'_H) &> n_L\pi(x_L) + n_H\pi(x_H), \\ U_L(x'_L) + V_L(x'_L) &= 0. \end{aligned}$$

To prove Lemma 10, fix a feasible schedule (M_L, x_L, M_H, x_H) (possibly $M_L = M_H$) such that $U_L(x_L) + V_L(x_L) > 0$. See Figure 4. (For simplicity, x_H is not shown in the figure.) Since $U_L(x_L) + V_L(x_L) > 0$, there exists $t' > 0$ such that $x'_L \equiv x_L + (0, t')$ satisfies

$$U_L(x'_L) + V_L(x'_L) = 0. \tag{30}$$

First, consider the case where $U_L(x_H) + V_L(x_H) \geq 0$. Then $\pi(x_H) \leq \pi(x^*_L)$ and $\pi(x_L) < \pi(x^*_L)$. Hence $(\{x^*_L\}, x^*_L, \{x^*_L\}, x^*_L)$ is a feasible schedule and generates more profits than (M_L, x_L, M_H, x_H). Since $U_L(x^*_L) + V_L(x^*_L) = 0$, we obtained the desired result.

Thus, in the remainder of the proof, assume $U_L(x_H) + V_L(x_H) < 0$. We now consider the following schedule:

$$(\{x'_L\}, x'_L, \{x_H\}, x_H).$$

Since x'_L satisfies (30) and $\pi(x'_L) - \pi(x_L) = t' > 0$, it remains to show that this schedule is feasible. By Lemma 3 (which is independent of Lemma 10), ex-ante IR is vacuous. By (30), ex-post IR holds for L. Since the initial schedule is feasible, ex-post IR also holds for H. Since $U_L(x_H) + V_L(x_H) < 0$ and the low types have downward temptation, $W_L(\{x_H\}) = 0$. This implies that ex-ante IC holds for L. It remains to show ex-ante IC for H, i.e., $W_H(\{x_H\}) \geq W_H(\{x'_L\})$. This is proved in the reminder of the proof.

First, $W_H(\{x'_L\})$ is given by

$$W_H(\{x'_L\}) = U_H(x'_L) + V_H(x'_L) - \max\{0, V_H(x'_L)\} = U_H(x'_L) + V_H(x'_L) \tag{31}$$

since $V_H \prec U_L + V_L$. On the other hand, since offers in $M_H \setminus \{x_H\}$ may be tempting,

$$W_H(\{x_H\}) \geq W_H(M_H). \tag{32}$$

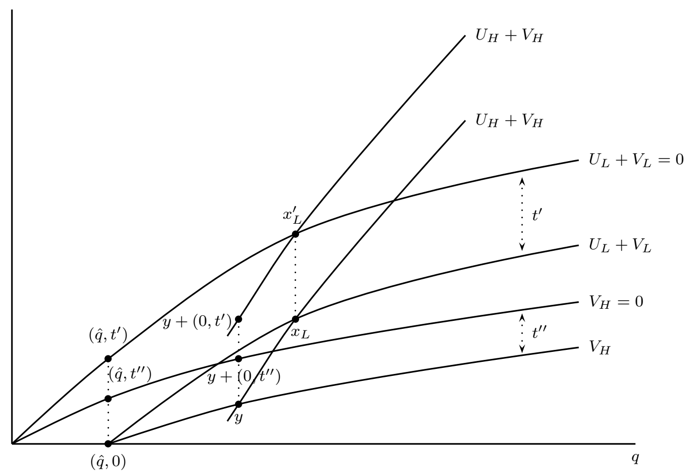


Figure 4: Proof of Lemma 2

Since the initial schedule satisfies ex-ante IC,

$$W_H(M_H) \geq W_H(M_L). \tag{33}$$

The ex-post IC for the low types in the initial schedule implies that none of the offers in M_L is below the indifference curve of $U_L + V_L$ that passes through x_L. Since $V_H \prec U_L + V_L$, no offer in M_L is more tempting for the high types than the offer $(\hat{q}, 0)$ defined by

$$U_L(\hat{q}, 0) + V_L(\hat{q}, 0) = U_L(x_L) + V_L(x_L). \tag{34}$$

That is,

$$V_H(\hat{q}, 0) \geq \max_{x \in M_L} V_H(x). \tag{35}$$

Since the right-hand side of (34) is strictly positive, $\hat{q} > 0$. By quasi-linearity, $U_L(\hat{q}, t') + V_L(\hat{q}, t') = 0$. Let $t'' > 0$ be defined by $V_H(\hat{q}, t'') = 0$. Since $V_H \prec U_L + V_L$, we have $t'' < t'$.

Let y be the offer defined by

$$V_H(y) = V_H(\hat{q}, 0), \tag{36}$$

$$U_H(y) + V_H(y) = U_H(x_L) + V_H(x_L). \tag{37}$$

Then

$$\begin{aligned}
W_H(M_L) &\geq U_H(x_L) + V_H(x_L) - V_H(\hat{q}, 0) && \text{by (35)} \\
&= U_H(y) + V_H(y) - V_H(y) && \text{by (36) and (37)} \\
&= U_H(y) + V_H(\hat{q}, t'') && \text{since } V_H(\hat{q}, t'') = 0 \\
&= U_H(y) + V_H(y + (0, t'')) && \text{by (36)} \\
&> U_H(y + (0, t')) + V_H(y + (0, t')) && \text{by } t' > t'' > 0 \\
&= U_H(x'_L) + V_H(x'_L) && \text{by (37).}
\end{aligned}$$

This, together with (31)–(33), implies $W_H(\{x_H\}) > W_H(\{x'_L\})$, as desired. Q.E.D.

A.5 Proof of Lemma 3

Let γ be any type with downward temptation. Let M be any menu and let $x \in M$ be such that $V_\gamma(x) \geq 0$. By downward temptation, $V_\gamma(x) \geq 0$ implies $U_\gamma(x) \geq 0$, and hence

$$U_\gamma(x) + V_\gamma(x) \geq V_\gamma(x),$$

which implies

$$\max_{y \in M} \left[U_\gamma(y) + V_\gamma(y)\right] \geq V_\gamma(x).$$

Since this holds for all $x \in M$ such that $V_\gamma(x) \geq 0$ and M includes $(0,0)$, we obtain

$$\max_{y \in M} \left[U_\gamma(y) + V_\gamma(y)\right] \geq \max_{y \in M} V_\gamma(y),$$

which means $W_\gamma(M) \geq 0$. Q.E.D.

A.6 Proof of Proposition 7

Let $(\{z, x_L\}, x_L, \{x_H\}, x_H)$ be as stated in the proposition. We first show that this schedule is feasible. For the low types, ex-post IC is trivial by construction. Ex-ante IR holds since $W_L(\{x_L, z\}) = U_L(z) \geq 0$. For ex-ante IC, it suffices to show that $U_L(x_H) < 0$. So, suppose $U_L(x_H) \geq 0$. Note that $U_L(x_L) \geq 0$. Since x_L^* maximizes π subject to $U_L \geq 0$, we have $\pi(x_L^*) \geq \max\{\pi(x_L), \pi(x_H)\}$. Thus, the pair $(z', x'_L) = (x_L^*, x_L^*)$ dominates (z, x_L) strictly since $\pi(X_H(x_L^*, x_L^*)) > \pi(x_L^*)$, which is a contradiction with the optimality of (z, x_L). For the high types, ex-post IC is trivial. Ex-ante IC holds since $W_H(\{x_H\}) = W_H(\{x_L, z\})$ by construction. For ex-ante IR, note $W_H(\{x_L, z\}) \geq U_H(z) + V_H(z) - V_H(z) = U_H(z)$. Since $U_L(z) \geq 0$, we have $U_H(z) \geq 0$.

To finish the proof, suppose that $(\{z, x_L\}, x_L, \{x_H\}, x_H)$ is not optimal. Thus it is dominated by another schedule (M'_L, x'_L, M'_H, x'_H). Since self-control costs are non-negative, $U_L(x'_L) \geq 0$. Let $z' = (q'_z, t'_z)$ be the offer defined by

$$U_L(z') + V_L(z') = U_L(x'_L) + V_L(x'_L),$$
$$\text{either } U_L(z') = 0 \text{ or } t'_z = 0.$$

We claim that $V_H(z') \geq \max_{x \in M'_L} V_H(x)$. If $t'_z = 0$, this follows from ex-post IC for L and strong downward temptation. If $U_L(z') = 0$, then ex-ante IR implies

$$0 \leq W_L(M'_L) = V_L(z') - \max_{x \in M'_L} V_L(x).$$

The inequality together with $U_L + V_L \succ V_H \succ V_L$ and ex-post IC for L yields the desired inequality.

The claim implies

$$W_H(M'_L) \geq U_H(x'_L) + V_H(x'_L) - V_H(z'). \tag{38}$$

Let $x''_L = x'_L$ if $x'_L \leq w_{z'}$ and $x''_L = w_{z'}$ otherwise. Then x''_L and x'_L lie on the same indifference curve of $U_L + V_L$. Since $U_L + V_L \prec U_H + V_H$, we obtain

$$U_H(x_L'') + V_H(x_L'') \leq U_H(x_L') + V_H(x_L'). \tag{39}$$

By ex-ante IC and non-negative self-control costs, $W_H(M_L') \leq W_H(M_H') \leq U_H(x_H')$. This, together with (38) and (39), implies

$$U_H(x_H') \geq U_H(x_L'') + V_H(x_L'') - V_H(z').$$

This implies $\pi(x_H') \leq \pi(x_H'')$, where $x_H'' = X_H(z', x_L'')$. Since $\pi(x_L'') \geq \pi(x_L')$, the pair (x_L'', x_H'') is at least as profitable as (x_L', x_H'), which is strictly more profitable than (x_L, x_H). Hence, the pair (z', x_L'') strictly dominates (z, x_L), a contradiction with the initial choice of (z, x_L). Q.E.D.

A.7 Uniqueness of Minimum Supporting Entry Fees

This section proves that for any feasible schedule, there exists a unique minimum supporting entry-fee profile. To show this, we prove the following property: for any feasible schedule (M_L, x_L, M_H, x_H), if $e = (e_L, e_H)$ and $e' = (e_L', e_H')$ are supporting entry-fee profiles, so is the coordinate-wise minimum $e^{min} \equiv (\min\{e_L, e_L'\}, \min\{e_H, e_H'\})$. This property implies the uniqueness since the set of supporting entry-fee profiles is a non-empty closed subset of $\mathbb{R}_+^2$.

So, we show that the schedule $(M_L \cup \{(0, e_L^{min})\}, x_L, M_H \cup \{(0, e_H^{min})\}, x_H)$ is feasible. For each type, ex-post IC and ex-ante IR are trivially satisfied since e and e' are supporting entry-fee profiles. Thus, it remains to prove ex-ante IC for each type. We prove only for the low type, since the proof for the high type works similarly. Suppose, by way of contradiction, that the low types prefer the menu intended for the high types, i.e.,

$$W_L\big(M_L \cup \{(0, e_L^{min})\}\big) < W_L\big(M_H \cup \{(0, e_H^{min})\}\big). \tag{40}$$

Since the left-hand side is non-negative by ex-ante IR, the right-hand side is positive. Assume, without loss of generality, that $e_H^{min} = e_H$. Then

$$W_L\big(M_H \cup \{(0, e_H)\}\big) > 0 \geq U_L(0, e_H) \geq U_L(0, e_H) + V_L(0, e_H) - \max_{M_H \cup \{(0, e_H)\}} V_L(x).$$

Looking at the both ends of these inequalities, we see that in the menu $M_H \cup \{(0, e_H)\}$, the offer $(0, e_H)$ does not maximize $U_L + V_L$. Thus

$$\begin{aligned} W_L\big(M_H \cup \{(0, e_H)\}\big) &= \max_{M_H}\big[U_L(x) + V_L(x)\big] - \max_{M_H \cup \{(0, e_H)\}} V_L(x) \\ &\leq \max_{M_H}\big[U_L(x) + V_L(x)\big] - \max_{M_H \cup \{(0, e_H')\}} V_L(x) \\ &\leq W_L\big(M_H \cup \{(0, e_H')\}\big). \end{aligned}$$

This and (40) imply that at least one of the following holds:

$$\begin{aligned} &W_L\big(M_L \cup \{(0, e_L')\}\big) < W_L\big(M_H \cup \{(0, e_H')\}\big), \text{ or} \\ &W_L\big(M_L \cup \{(0, e_L)\}\big) < W_L\big(M_H \cup \{(0, e_H)\}\big). \end{aligned}$$

This is a contradiction since e and e' are both supporting entry-fee profiles. Q.E.D.

A.8 Proof of Proposition 8

We first prove the following result, which holds for any feasible schedule:

Lemma 11. *For any feasible schedule, $(\hat{e}_L, \hat{e}_H)$ is a supporting entry-fee profile.*

Proof. Let (M_L, x_L, M_H, x_H) be any feasible schedule. We show that the modified schedule $(M_L \cup \{(0, \hat{e}_L)\}, x_L, M_H \cup \{(0, \hat{e}_H)\}, x_H)$ is also feasible. So, take any type $\gamma \in \{L, H\}$. The following proof works in the same way for each type.

(Ex-post IC) We first show that the modified schedule satisfies ex-post IC for γ. Since the initial schedule is feasible, it suffices to show that the added item $(0, \hat{e}_\gamma)$ is not strictly preferred to x_γ by $U_\gamma + V_\gamma$. To show this, first consider the case where type γ has upward temptation. Since the initial schedule is feasible,

$$0 \leq W_\gamma(M_\gamma) = U_\gamma(x_\gamma) + V_\gamma(x_\gamma) - \max_{x \in M_\gamma} V_\gamma(x) \leq U_\gamma(x_\gamma).$$

Then, since $U_\gamma + V_\gamma \succ U_\gamma$, we obtain $U_\gamma(x_\gamma) + V_\gamma(x_\gamma) \geq 0$. Since $U_\gamma(0, \hat{e}_\gamma) + V_\gamma(0, \hat{e}_\gamma) \leq 0$, we have $U_\gamma(x_\gamma) + V_\gamma(x_\gamma) \geq U_\gamma(0, \hat{e}_\gamma) + V_\gamma(0, \hat{e}_\gamma)$, as desired.

Now, consider the case when γ has downward temptation. Suppose, by way of contradiction,

$$U_\gamma(0, \hat{e}_\gamma) + V_\gamma(0, \hat{e}_\gamma) > U_\gamma(x_\gamma) + V_\gamma(x_\gamma). \tag{41}$$

Then there exists $e'_\gamma > \hat{e}_\gamma$ such that $U_\gamma(0, e'_\gamma) + V_\gamma(0, e'_\gamma)$ equals the right-hand side of (41). Since x_γ satisfies ex-post IC in the initial schedule, for all $x \in M_\gamma$,

$$U_\gamma(x) + V_\gamma(x) \leq U_\gamma(0, e'_\gamma) + V_\gamma(0, e'_\gamma).$$

Since $V_\gamma \prec U_\gamma + V_\gamma$, it follows that for all $x \in M_\gamma$, $V_\gamma(x) \leq V_\gamma(0, e'_\gamma)$. But then

$$\max_{x \in M_\gamma} V_\gamma(x) \leq V_\gamma(0, e'_\gamma) < V_\gamma(0, \hat{e}_\gamma) = \min\{0, \max_{x \in M_\gamma} V_\gamma(x)\},$$

which is a contradiction.

(Ex-ante IR) Ex-post IC just proved and $V_\gamma(0, \hat{e}_\gamma) \leq \max_{x \in M_\gamma} V_\gamma(x)$ imply

$$\begin{aligned} W_\gamma(M_\gamma \cup \{(0, \hat{e}_\gamma)\}) &= U_\gamma(x_\gamma) + V_\gamma(x_\gamma) - \max_{M_\gamma \cup \{(0, \hat{e}_\gamma)\}} V_\gamma(x) \\ &= U_\gamma(x_\gamma) + V_\gamma(x_\gamma) - \max_{M_\gamma} V_\gamma(x) = W_\gamma(M_\gamma) \geq 0. \end{aligned} \tag{42}$$

(Ex-ante IC) Suppose, by way of contradiction, that for $\theta \neq \gamma$, $W_\gamma(M_\gamma \cup \{(0, \hat{e}_\gamma)\}) < W_\gamma(M_\theta \cup$

$\{(0,\hat{e}_\theta)\})$. Then

$$\begin{aligned}
&\max_{M_\theta}\left[U_\gamma(x)+V_\gamma(x)\right]-\max_{M_\theta}V_\gamma(x)\\
&\qquad = W_\gamma(M_\theta) \le W_\gamma(M_\gamma)\\
&\qquad = W_\gamma(M_\gamma \cup \{(0,\hat{e}_\gamma)\}) \qquad \text{by (42)}\\
&\qquad < W_\gamma(M_\theta \cup \{(0,\hat{e}_\theta)\})\\
&\qquad = \max_{M_\theta\cup\{(0,\hat{e}_\theta)\}}\left[U_\gamma(x)+V_\gamma(x)\right]-\max_{M_\theta\cup\{(0,\hat{e}_\theta)\}}V_\gamma(x)\\
&\qquad \le \max_{M_\theta\cup\{(0,\hat{e}_\theta)\}}\left[U_\gamma(x)+V_\gamma(x)\right]-\max_{M_\theta}V_\gamma(x).
\end{aligned} \tag{43}$$

Looking at the both ends of these inequalities, we see that $(0,\hat{e}_\theta)$ maximizes $U_\gamma+V_\gamma$ in the menu $M_\theta\cup\{(0,\hat{e}_\theta)\}$. Thus,

$$\begin{aligned}
W_\gamma(M_\theta\cup\{(0,\hat{e}_\theta)\}) &= U_\gamma(0,\hat{e}_\theta)+V_\gamma(0,\hat{e}_\theta)-\max_{M_\theta\cup\{(0,\hat{e}_\theta)\}}V_\gamma(x)\\
&\le U_\gamma(0,\hat{e}_\theta)\le 0.
\end{aligned}$$

But then, (43) implies $W_\gamma(M_\gamma)<0$. This is a contradiction since the initial schedule (M_L,x_L,M_H,x_H) is feasible. Q.E.D.

Lemma 12. *Let (M_L,x_L,M_H,x_H) be an optimal schedule.*

(i) If the high types have upward temptation, $W_L(M_L)=0$.

(ii) If the high types have weak downward temptation, $W_L(M_L)=0$ and $W_H(M_H)=W_H(M_L)$.

(iii) If the high types have strong downward temptation and the schedule is the one in Proposition 7, then $W_H(M_H)=W_H(M_L)$. In addition, if $t_z>0$, then $W_L(M_L)=0$.

Proof. (i) This is trivial since perfect discrimination is possible.

(ii) Since (x_L,x_H) is a solution to the standard problem with U_γ, (19) and (20) bind. The proof for those inequalities then implies $W_L(M_L)=0$ and $W_H(M_H)=W_H(M_L)$.

(iii) By the construction of the schedule, $W_H(M_H)=W_H(M_L)$. If $t_z>0$, then $W_L(M_L)=U_L(z)=0$. Q.E.D.

The following lemma completes the proof of Proposition 8.

Lemma 13. *For any optimal schedule (M_L,x_L,M_H,x_H) in Propositions 5–7, if a pair (e_L,e_H) is a supporting entry-fee profile for the schedule, then $e_L\ge\hat{e}_L$ and $e_H\ge\hat{e}_H$.*

Proof. To prove $e_L\ge\hat{e}_L$, suppose, by way of contradiction, that $\hat{e}_L>e_L\ge 0$. Then $\max_{M_L}V_L(x)=V_L(0,\hat{e}_L)<V_L(0,e_L)$. Thus

$$\begin{aligned}
W_L(M_L\cup\{(0,e_L)\}) &= U_L(x_L)+V_L(x_L)-\max_{M_L\cup\{(0,e_L)\}}V_L(x)\\
&= U_L(x_L)+V_L(x_L)-V_L(0,e_L)\\
&< U_L(x_L)+V_L(x_L)-V_L(0,\hat{e}_L)=W_L(M_L)=0,
\end{aligned} \tag{44}$$

where the last equality follows from Lemma 12 and $\hat{e}_L > 0$; note that if the high types have strong downward temptation and $t_z = 0$, then $\hat{e}_L = 0$. By (44), $M_L \cup \{(0,e_L)\}$ violates ex-ante IR, which is a contradiction since (e_L, e_H) is a supporting entry-fee profile.

To prove $e_H \geq \hat{e}_H$, suppose, by contradiction, that $\hat{e}_H > e_H \geq 0$. Then, as in the previous paragraph,

$$\begin{aligned} W_H(M_H \cup \{(0,e_H)\}) &= U_H(x_H) + V_H(x_H) - \max_{M_H \cup \{(0,e_H)\}} V_H(x) \\ &< W_H(M_H) = W_H(M_L), \end{aligned} \tag{45}$$

where the last equality follows from Lemma 12 and $\hat{e}_H > 0$; note that if the high types have upward temptation, then $x^*_H \in M_H$ and hence $\hat{e}_H = 0$.

We now claim

$$V_H(0,e_L) \leq \max_{x \in M_L} V_H(x). \tag{46}$$

Suppose that this does not hold. Then there exists $e'_L > e_L$ such that $V_H(0,e'_L)$ equals the right-hand side of (46). Thus, for all $x \in M_L$, $V_H(x) \leq V_H(0,e'_L)$. Since $V_L \prec V_H$, we obtain that for all $x \in M_L$, $V_L(x) \leq V_L(0,e'_L)$. But then

$$\max_{x \in M_L} V_L(x) \leq V_L(0,e'_L) < V_L(0,e_L) \leq V_L(0,\hat{e}_L) \leq \max_{x \in M_L} V_L(x).$$

This contradiction proves (46).

By (46),

$$W_H(M_L \cup \{(0,e_L)\}) = \max_{M_L \cup \{(0,e_L)\}} \big[U_H(x) + V_H(x)\big] - \max_{M_L} V_H(x) \geq W_H(M_L).$$

This and (45) imply $W_H(M_H \cup \{(0,e_H)\}) < W_H(M_L \cup \{(0,e_L)\})$. Thus ex-ante IC is violated, which is a contradiction since (e_L, e_H) is a supporting entry-fee profile. Q.E.D.

A.9 Proof of Proposition 9

Let (M_L, x_L, M_H, x_H) be any feasible schedule. By Lemma 11 in Appendix A.8, $(\hat{e}_L, \hat{e}_H)$ is a supporting entry-fee profile. Let $\gamma \in \{L,H\}$ be a type that has upward temptation. By ex-ante IR and ex-post IC, $U_\gamma(x_\gamma) \geq 0$. Since $V_\gamma \succ U_\gamma$, it follows that $V_\gamma(x_\gamma) \geq 0$. Thus $\hat{e}_\gamma = 0$. Then, if (e^*_L, e^*_H) is the minimum supporting entry-fee profile, $e^*_\gamma \leq \hat{e}_\gamma = 0$, hence $e^*_\gamma = 0$. Q.E.D.

A.10 Many Types

We here generalize Propositions 1 and 5 to many types. Let $\Gamma = \{1,2,\ldots,T\}$ be a finite set of types. Let n_γ denote the fraction of consumers whose type is γ (thus $\sum_{\gamma \in \Gamma} n_\gamma = 1$). It is straightforward to generalize Assumptions 1–6 and other definitions to many types.

Proposition 10. *Assume that each type has upward temptation. Then, whether entry fees are feasible or not, there exists a feasible schedule $(M_\gamma, x_\gamma)_{\gamma \in \Gamma}$ such that $x_\gamma = x^*_\gamma$ for each $\gamma \in \Gamma$.*

Proof. It suffices to consider the case where entry fees are not feasible (note that x^*_γ is not affected by whether entry fees are feasible). To construct a desired schedule, fix a type $\theta \in \Gamma$. We construct a sequence of menus $(M_\theta^\gamma)_{\gamma=\theta}^T$ such that all the menus are finite sets and $M_\theta^\gamma \subseteq M_\theta^{\gamma+1}$ for all γ. The first menu is $M_\theta^\theta \equiv \{x_\theta^*\}$. To specify the remainder, let $\gamma > \theta$ and assume that we have specified finite menus up to $M_\theta^{\gamma-1}$. Let y_θ^γ be the offer such that

$$\begin{aligned} U_\gamma(y_\theta^\gamma) + V_\gamma(y_\theta^\gamma) &= \max_{M_\theta^{\gamma-1}} \big[U_\gamma(x) + V_\gamma(x)\big], \\ U_\gamma(y_\theta^\gamma) &= 0. \end{aligned}$$

The offer y_θ^γ is well-defined since $U_\gamma + V_\gamma \succ U_\gamma$ and the right-hand side of the first line is positive (because $U_\gamma(x_\theta^*) + V_\gamma(x_\theta^*) > 0$).

Now, if $\max_{M_\theta^{\gamma-1}} V_\gamma(x) \geq V_\gamma(y_\theta^\gamma)$, then we set $M_\theta^\gamma = M_\theta^{\gamma-1}$. If not, let z_θ^γ be the offer defined by

$$\begin{aligned} V_\gamma(z_\theta^\gamma) &= V_\gamma(y_\theta^\gamma), \\ V_\theta(z_\theta^\gamma) &= \min\{V_\theta(y_\theta^\gamma), V_\theta(x_\theta^*)\}, \end{aligned} \tag{47}$$

and set $M_\theta^\gamma = M_\theta^{\gamma-1} \cup \{z_\theta^\gamma\}$. Since $V_\theta \prec V_\gamma$ and z_θ^γ and y_θ^γ are on the same indifference curve of V_γ, we have $z_\theta^\gamma \geq y_\theta^\gamma$. This and $U_\gamma + V_\gamma \prec V_\gamma$ imply

$$U_\gamma(z_\theta^\gamma) + V_\gamma(z_\theta^\gamma) \leq U_\gamma(y_\theta^\gamma) + V_\gamma(y_\theta^\gamma) = \max_{M_\theta^{\gamma-1}} \big[U_\gamma(x) + V_\gamma(x)\big].$$

Hence,

$$\begin{aligned} W_\gamma(M_\theta^\gamma) &= \max_{M_\theta^\gamma} \big[U_\gamma(x) + V_\gamma(x)\big] - \max_{M_\theta^\gamma} V_\gamma(x) \\ &= \max_{M_\theta^{\gamma-1}} \big[U_\gamma(x) + V_\gamma(x)\big] - \max\{V_\gamma(y_\theta^\gamma), \max_{M_\theta^{\gamma-1}} V_\gamma(x)\} \\ &\leq U_\gamma(y_\theta^\gamma) + V_\gamma(y_\theta^\gamma) - V_\gamma(y_\theta^\gamma) = 0. \end{aligned}$$

Given $(M_\theta^\gamma)_{\gamma=\theta}^T$, let $M_\theta \equiv M_\theta^T$. We claim that $(M_\theta, x_\theta^*)_{\theta\in\Gamma}$ is a feasible schedule.

We begin by proving ex-ante IC for the "downward" direction. To this end, we first show that for any triple $k > \gamma \geq \theta$,

$$\max_{M_\theta^k} \big[U_\gamma(x) + V_\gamma(x)\big] = \max_{M_\theta^{k-1}} \big[U_\gamma(x) + V_\gamma(x)\big]. \tag{48}$$

To see this, assume $M_\theta^k \neq M_\theta^{k-1}$ (otherwise, there is nothing to prove). Let w be such that

$$w \in \operatorname*{Argmax}_{M_\theta^{k-1}} \big[U_k(x) + V_k(x)\big].$$

Since $M_\theta^k \neq M_\theta^{k-1}$,

$$V_k(y_\theta^k) > \max_{M_\theta^{k-1}} V_k(x) \geq V_k(w).$$

Since $V_k \succ U_k + V_k$, and w and y_θ^k are on the same indifference curve of $U_k + V_k$, we have $y_\theta^k \gg w$.

Recall $z^k_\theta \geq y^k_\theta$ and they lie on the same indifference curve of V_k. Since $V_k \succ U_k + V_k \succ U_\gamma + V_\gamma$,

$$\max_{M^{k-1}_\theta}\big[U_\gamma(x)+V_\gamma(x)\big] \geq U_\gamma(w)+V_\gamma(w) > U_\gamma(y^k_\theta)+V_\gamma(y^k_\theta) \geq U_\gamma(z^k_\theta)+V_\gamma(z^k_\theta).$$

This and $M^k_\theta = M^{k-1}_\theta \cup \{z^k_\theta\}$ imply (48).

(48) implies that for any pair $\gamma \geq \theta$,

$$\max_{M_\theta}\big[U_\gamma(x)+V_\gamma(x)\big] = \max_{M^\gamma_\theta}\big[U_\gamma(x)+V_\gamma(x)\big]. \tag{49}$$

Therefore, for any pair $\gamma \geq \theta$,

$$\begin{aligned} W_\gamma(M_\theta) &= \max_{M^\gamma_\theta}\big[U_\gamma(x)+V_\gamma(x)\big] - \max_{M_\theta} V_\gamma(x) \\ &\leq \max_{M^\gamma_\theta}\big[U_\gamma(x)+V_\gamma(x)\big] - \max_{M^\gamma_\theta} V_\gamma(x) = W_\gamma(M^\gamma_\theta) \leq 0. \end{aligned}$$

Thus, consumers have no ex-ante incentive to mimic lower types, i.e., ex-ante IC is satisfied for the downward direction.

Since (49) also holds for $\gamma = \theta$, ex-post IC is satisfied. By construction, for all $x \in M_\theta$, $x \geq x^*_\theta$. This and ex-post IC imply

$$U_\theta(x) \leq 0 \quad \text{for all } x \in M_\theta. \tag{50}$$

This implies that for any pair $\gamma < \theta$, $W_\gamma(M_\theta) \leq 0$. This implies that ex-ante IC is also satisfied for the upward direction.

By (47),

$$\max_{M_\theta} V_\theta(x) = V_\theta(x^*_\theta).$$

This and ex-post IC, $W_\theta(M_\theta) = U_\theta(x^*_\theta) + V_\theta(x^*_\theta) - V_\theta(x^*_\theta) = 0$, hence ex-ante IR is also satisfied. This proves that $(M_\theta, x^*_\theta)_{\theta\in\Gamma}$ is a feasible schedule. Q.E.D.

If not all types have upward temptation, the seller can still achieve perfect discrimination for an upper end of consumers, as the following corollary shows.

Corollary 1. *Let $\hat{\gamma} \in \Gamma$ be such that all types $\gamma \geq \hat{\gamma}$ have upward temptation. Then there exists a feasible schedule that earns $\sum_{\gamma\geq\hat{\gamma}} n_\gamma \pi(x^*_\gamma)$.*

Proof. Use the construction of Proposition 10 to the types $\gamma \geq \hat{\gamma}$, ignoring all types $\gamma < \hat{\gamma}$. We then obtain a list of menus $(M_\gamma)_{\gamma\geq\hat{\gamma}}$. If the seller offers these menus, by Proposition 10, each type $\gamma \geq \hat{\gamma}$ has an incentive to choose M_γ and then x^*_γ. By (50), for any pair γ, θ such that $\gamma < \hat{\gamma} \leq \theta$, we have $U_\gamma(x) \leq 0$ for all $x \in M_\theta$ and hence $W_\gamma(M_\theta) \leq 0$. This implies that types $\gamma < \hat{\gamma}$ have no incentive to choose any of the menus offered by the seller. Therefore, a schedule $(\hat{M}_\gamma, \hat{x}_\gamma)_{\gamma\in\Gamma}$ defined by $(\hat{M}_\gamma, \hat{x}_\gamma) = (\{(0,0)\}, (0,0))$ for all $\gamma < \hat{\gamma}$ and $(\hat{M}_\gamma, \hat{x}_\gamma) = (M_\gamma, x^*_\gamma)$ for all $\gamma \geq \hat{\gamma}$, is feasible. Q.E.D.

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